UNIVERSITY OF CALGARY
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Investments & Portfolio Management

APT and Multifactor Models

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Academics do not like the single-index CAPM much now, but practitioners still find it useful.

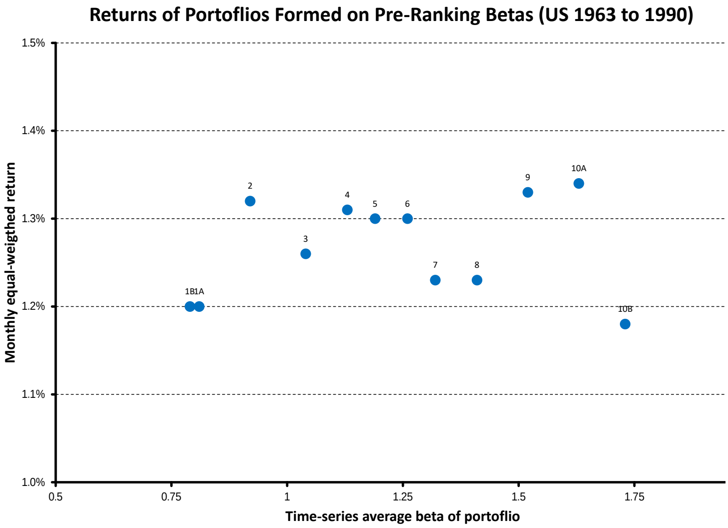
- While it is difficult to conclusively test CAPM, the model is afflicted by many 'problems', has failed many tests, and its predictions are often just not observed in the data (e.g. portfolios with higher betas not being correlated with higher returns, see next slide).

By and large academics agree that the 'market' is an useful explanatory variable, so they have developed and tested various models using additional explanatory variables (aka 'factors').

- If multiple sources of systematic risk are deemed to exist, why expect all of them to be captured by a single factor rather than several?

This said, it seems as if practitioners are not finding these new models significantly more useful than the CAPM in term of pricing individual securities.

However, portfolio managers have developed all kind of active investment strategies to attempt harvesting additional returns from a wide variety of factors (aka factor investing).


Returns of Portoflios Formed on Pre-Ranking Betas (US 1963 to 1990)
Monthly equal-weigthed return
1.5%
1.4%
1.3%
1.2%
1.1%
1.0%
1B
1A
2
3
4
5
6
7
8
9
10A
10B
0.5
0.75
1
1.25
1.5
1.75
Time-series average beta of portoflio

Assumptions underlying the APT

- Security returns can be described by a factor model (one or several factors);
- There are sufficient securities to diversify away idiosyncratic risk (well-diversified portfolios have security weights w_i small enough that non-systematic variance is negligible).
- Well-functioning security markets do not allow for the persistence of arbitrage opportunities (i.e. the law of one price is enforced by arbitrageurs), leading to the no arbitrage condition (i.e. arbitrage opportunities disappear as soon as they appear and therefore are assumed not to exist).

In a CAPM world, all investors tilt a little bit their mean-variance efficient portfolios toward the underpriced security or away from an overpriced security, and price correction ensue.

In an APT world, arbitrageurs achieve the same price correction outcome in an instant. However, it has been argued that there are limits to arbitrage, so the no arbitrage condition remains an assumption.

The academics who developed the APT suggested the factors ought to be unexpected changes in macro-economic variables influencing all firms at once which cannot be diversified away.

- However, many key macro-economic variable like inflation or GNP are not available in real time, are released at best monthly with error, and are often revised;
- In addition, a forecast model is required for each macro-economic variable in order to extract the unexpected component of the change in the variable upon announcement;
- So, implementing the APT as originally conceived requires using multi data sources to recreate, proxy or calculate the unexpected changes in several macro-economic variables;
- The above explains why practitioners do not bother much with the APT as originally conceived, as they find it too cumbersome to implement.

The APT nevertheless provided theoretical and methodological foundations for further development of multi-factor asset pricing models.

- A factor portfolio is well-diversified and has a beta of one for one factor and zero beta for all other factors. It tracks the returns induced by one factor uninfluenced by other factors.

The Fama-French three-factor model (1992)

The Fama-French three-factor model use as factors: the market, 'small versus big' firms, and 'high book to market (value) versus small book to market (growth)' firms. The key idea is to use some readily available firm characteristics to proxy exposure to systematic risk factors. The time-series of these factors are available on Kenneth French website.

$$E\widetilde{R}_i = \alpha_i + r_f + \beta_{iM}\left(E\widetilde{R}_M - r_f\right) + \beta_{iSMB}SMB + \beta_{iHML}HML$$

- α_i: abnormal return of security i ($E\alpha_i = 0$ assuming model fully explains returns)
- $\widetilde{R}_i$: return of security i (a random variable)
- $\widetilde{R}_m$: market return (a random variable)
- r_f: risk free rate of return
- SMB : Small Minus Big, the small size premium (R of small firms $>$ R of large firms)
- HML : High Minus Low, the value premium (R of high B/M firms $>$ R of low B/M firms)
- β_i: beta of security i for each risk factor
- E: expectation operator (expected value of a random variable, a mean)

The Fama-French three-factor model (1992)

Small minus big factor (SMB).

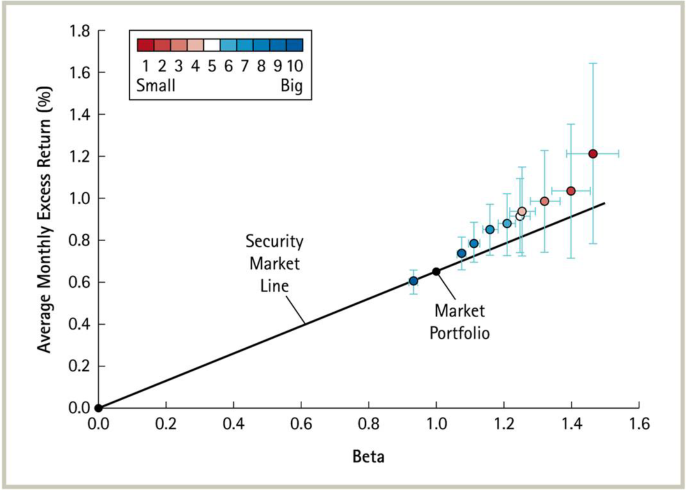
- The size premium (SMB) is the average monthly return on the smallest 30% of stocks (by market capitalization) minus the average monthly return on the largest 30%.
- When small stocks do well relative to large stocks, this will be positive; when they do worse than large stocks, this will be negative.

High minus low factor (HML)

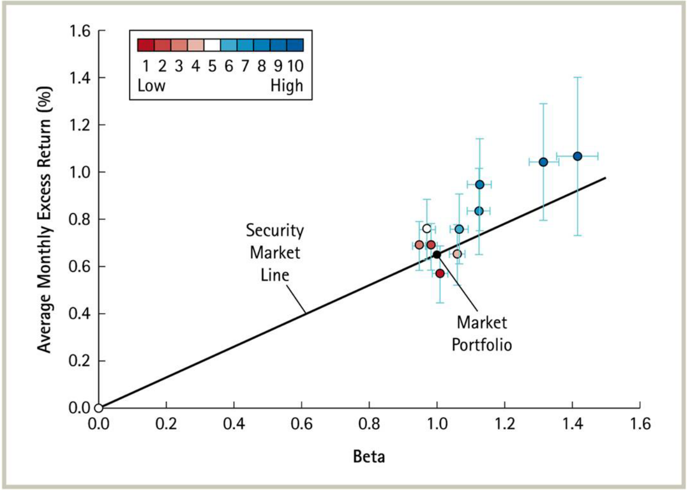
- The value premium (HML) is the average monthly return for the 50% of stocks with the highest book-to-market ratio minus the average return for the 50% of stocks with the lowest book-to-market ratio.
- When high value stocks do well relative to low value stocks, this will be positive; when they do worse than low value stocks, this will be negative.
- High book-to-market stocks are considered "value" stocks; low book-to-market stocks are considered "growth" stocks.

A factor beta (such as those above) is the sensitivity of security's returns to a particular systematic risk as proxied by a factor, and can be either positive or negative.

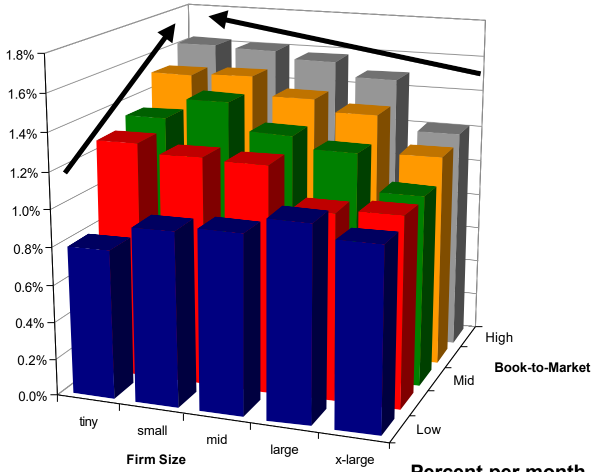
Evidence in support of the SMB factor



Evidence in support of the MHL factor

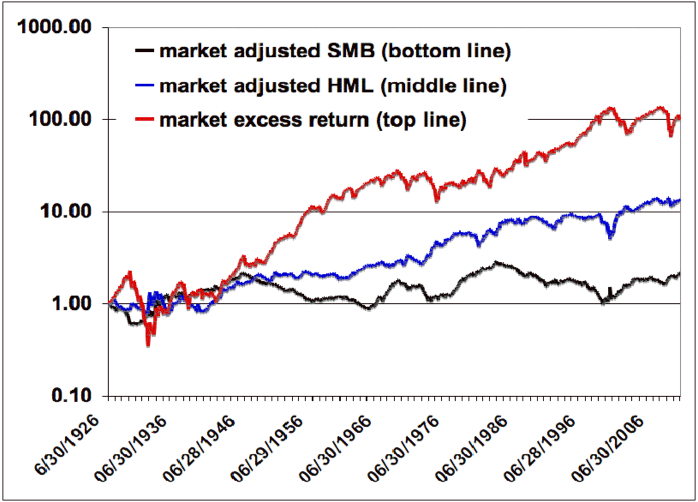


Joint influence of the SMB and MHL factors (US firms)



Source: Mertens, Data from Fama and French (1992)

Relative influence over time of the Fama-French three factors



Carhart fourth factor: Momentum (1997)

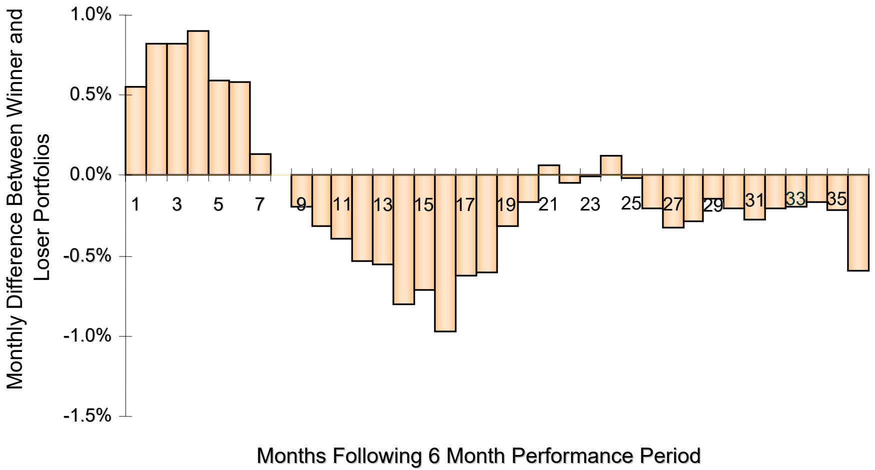
Persistence of returns, aka 'momentum' (winners minus losers: WML).

- It has been observed that stock returns tend to persist over several months but eventually die out, a sort of short term momentum property of returns.
- The factor is constructed in a similar fashion as SMB and HML by sorting stocks in two portfolios of high returns and low returns realized over the last few months.
- The so-called four-factor model comprises the original Fama-French three-factor augmented with a momentum factor.

Interpretation of SMB, HML, and WML

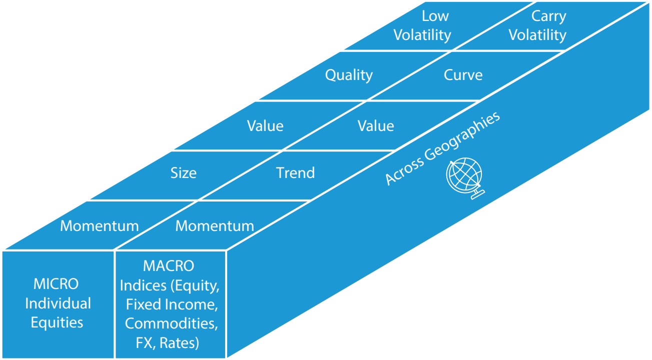
- In contrast to the APT factors, SMB, HML, and WML are 'empirical' factors (i.e. not derived from economic theory, but from found from analyzing empirical data).
- Such empirical factors might be proxies for extra-market sources of risk, or 'quirks' of securities markets (AKA 'anomalies'), or the by-product of market randomness.
- As many other factors in addition to SMB, HML, and WHL have been found to seemingly help explain and predict returns, the significance of such factors is not easy to assess.

Evidence in support of the WML factor



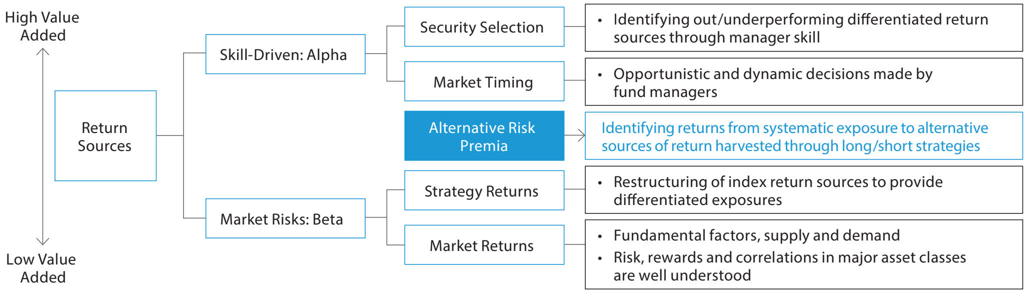
Source: Brunnermeier

Commonly used risk premia



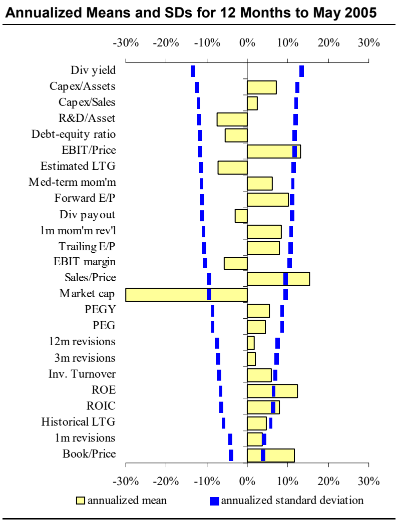
Source: Morgan Stanley Investment Management

How ‘factor investing’ fits in portfolio management

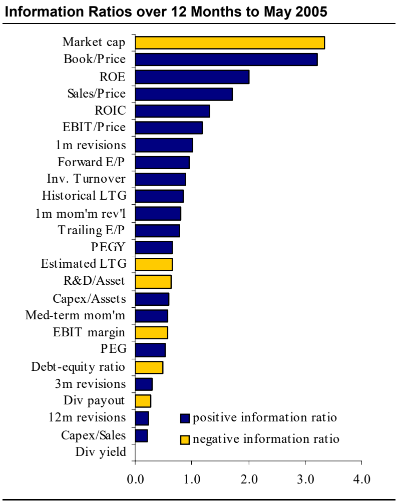


Source: Morgan Stanley Investment Management

Some factors seem somewhat reliable, but most seem marginal



Source: Morgan Stanley Quantitative Strategy. Data as of May 31, 2005.



Source: Morgan Stanley Quantitative Strategy. Data as of May 31, 2005.

Applications in chapter 10 of the textbook

Concept checks

- Suggest to do concept checks 1 to 3 (solutions provided at the end of the chapter).

Exercises

- Suggest 10-4, 10-5, and 10-11.
- Solutions follow next slides, and Excel solution file in available in D2L.

Portfolio	Beta1	Beta2	ER
A	1.5	2.0	31%
B	2.2	-0.2	27%

$E(r_p) = r_f + B_{p1}[E(r_1) - r_f] + B_{p2}[E(r_2) - r_f]$

.31 = .06 + 1.5 x RP1 + 2.0 x RP2

.27 = .06 + 2.2 x RP1 + (–.2) x RP2

By substitution: RP1=10% and RP2=5%

$E(r_p) = 6\% + B_{p1} \times 10\% + B_{p2} \times 5\%$

10-5 Solution

Portfolio	ER	Beta
A	12%	1.2
F	6%	0.0
E	8%	0.6

The return of portfolio F equals the risk-free rate since its beta equals 0.
Portfolio A's ratio of risk premium to beta is: (.12 - .06)/1.2 = .05,
whereas, portfolio E's ratio is lower at (.08 – .06)/.6 = .0333.
Create a portfolio G with beta equal to .6 (the same as E's) by mixing portfolios A and F in equal weights.

BG = .5 x 1.2 + .5 x 0 = 0.6

E(rG) = .5 x 12% + .5 x 6% = 9%

Buy G and sell E in equal amount to pocket 1% risk-free.

rG – rE = (9% + .6 x RM) – (8% + .6 x RM) = 1%

Factor	Risk prem.
I	6%
R	2%
C	4%
Rf	6%

r = 15% + (1 × I) + (0.5 × R) + (0.75 × C) + e

required E(r) = 6% + (1 × 6%) + (0.5 × 2%) + (0.75 × 4%) = 16%

Because the actually expected return based on risk is less than the equilibrium return, we conclude that the stock is overpriced.