UNIVERSITY OF CALGARY
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Investments & Portfolio Management

Fixed Income Portfolios

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For an equity portfolio, its CAPM β is a measure of the sensitivity of its returns to the returns of the market (i.e. the higher the β, the higher deemed risk and return of the porfoltio).

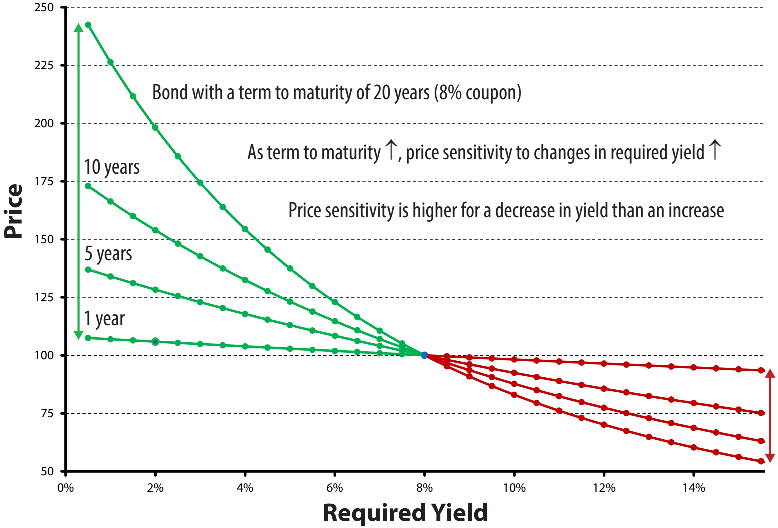
How can we measure the price sensitivity of a bond (or a portfolio of fixed-income securities) to changes in interest rates? (since change in interest rates is the key driver of bonds' prices and therefore their holding period returns). Duration and convexity are the two metrics used (often jointly).

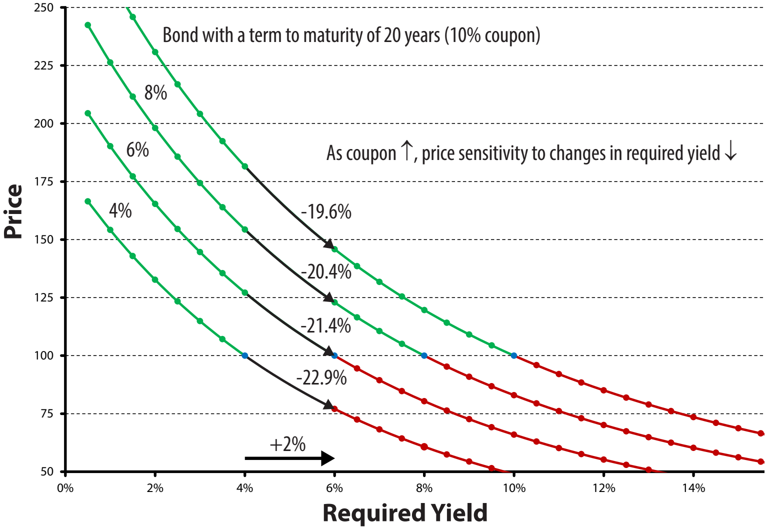
Key apparent drivers of interest rate sensitivity/risk.

- Bond prices and yields are inversely related.
- Long-term bonds tend to be more price sensitive than short-term bonds.
  - As maturity increases, price sensitivity increases at a decreasing rate.
- An increase in yield leads to a smaller price change than a decrease of equal magnitude.
  - Price sensitivity is inversely related to the yield to maturity at which the bond is selling.
- Interest rate risk is inversely related to the bond's coupon rate.

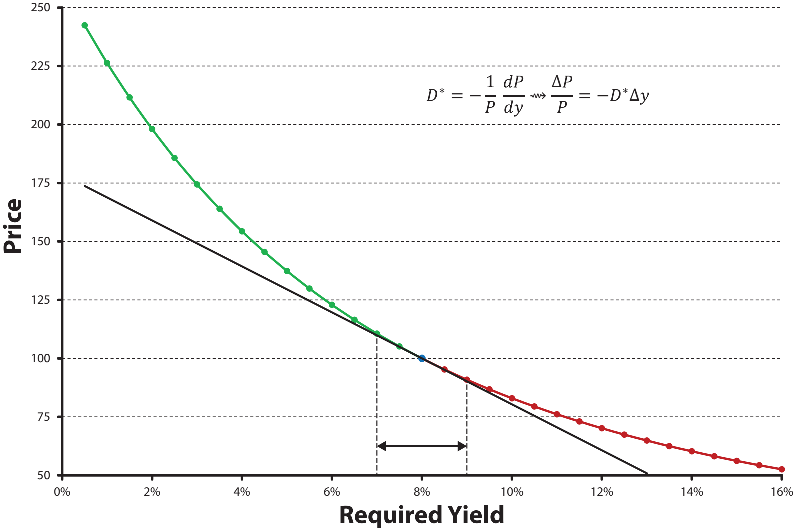
Useful to match the interest rate sensitivity of assets and liabilities, and 'immunize' the balance sheet.

Bond price volatility versus maturity as a function of changes in yield




Bond with a term to maturity of 20 years (10% coupon)
8%
6%
4%
As coupon ↑, price sensitivity to changes in required yield ↓
-19.6%
-20.4%
-21.4%
-22.9%
+2%
Price
250
225
200
175
150
125
100
75
50
0%
2%
4%
6%
8%
10%
12%
14%
Required Yield

Duration: a local first-degree estimator (i.e. ~ slope of tangent)



$$D^* = -\frac{1}{P}\frac{dP}{dy} \rightsquigarrow \frac{\Delta P}{P} = -D^*\Delta y$$

Macaulay's duration captures the concept of 'effective maturity': it is a measure of the weighted average time to maturity of the bond's cash flows.

Macaulay's duration (time measure with units in years; see textbook spreadsheet 16.1)

$$D = \frac{1}{P_0} \sum_{t=1}^{T} t \frac{CF_t}{(1+y)^t}$$

What is the Macaulay duration of a 3-year bond with an annual coupon of 3% for a yield to maturity of 3% and a market price of 100?

$$\begin{aligned} D &= \frac{1}{100}\left[1 \times \frac{3}{(1+.03)^1} + 2 \times \frac{3}{(1+.03)^2} + 3 \times \frac{100+3}{(1+.03)^3}\right] \\ &= \frac{1}{100}\left[1 \times 2.913 + 2 \times 2.828 + 3 \times 94.260\right] = \frac{291.347}{100} = 2.913 \; \mathit{years} \end{aligned}$$

Modified Duration (linear estimate of the percentage rate of price change with respect to yield)

- Adjust the Macaulay Duration by the factor $\frac{1}{(1+i)} \rightarrow D^* = \frac{D}{1+y}$. It measures the approximate percentage change in a bond's price for a 1% change in yield.

$$\frac{\triangle P}{P} = -D\left[\frac{\triangle(1+y)}{1+y}\right] = -\frac{D}{1+y}\triangle y = -D^*\triangle y \rightarrow P_{y+\triangle y} = P_y \times (1 - D^* \times \triangle y)$$

Using the modified duration, calculate the expected price of a a 3-year bond with an annual coupon of 3%, a yield to maturity of 3% and a market price of 100, assuming a uniform increase in interest rates of 0.1%.

$$P_{3.1\%} = P_{3.0\%} \times (1 - D^* \times \triangle y) = P_{3.0\%} \times \left(1 - \frac{D}{1+y} \times \triangle y\right)$$

$$= 100 \times \left(1 - \frac{2.913}{1+.03} \times 0.001\right) = 99.72$$

Understanding Duration

While for a zero-coupon bond its duration equals its maturity, a coupon-paying bond has a lower duration as its coupon rate is higher (holding maturity constant).

- The higher the coupon rate, the more significant the cash flows prior to maturity, therefore contributing to a lower duration.

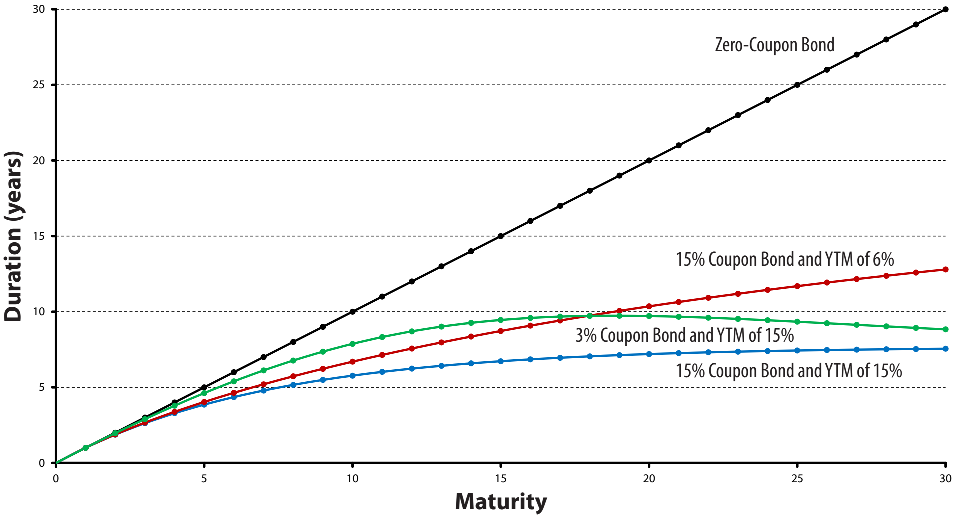
Holding the coupon rate constant, a bond's duration generally increases with its time to maturity (always for a bond selling at par or at a premium).

Other factors constant, the duration of a coupon bond is higher when its YTM is lower.

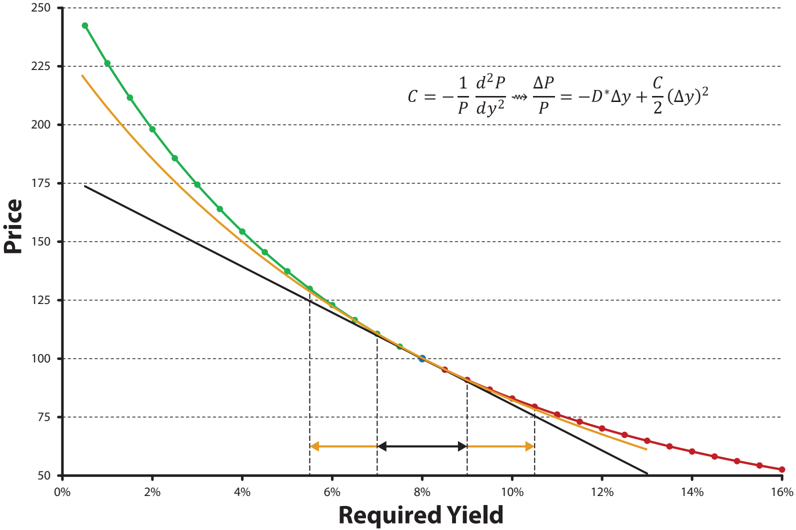
- The lower the yield, the higher the PV of later cash flows, leading to a higher duration.

The duration of a perpetuity equals $\frac{1+y}{y}$.

The duration of a fixed-income portfolio can be estimated either using all expected cash flows or a weighted average of the duration of the constituent parts of the portfolio $D_p = \sum w_i D_i$.


Duration (years)
30
25
20
15
10
5
0
Maturity
0
5
10
15
20
25
30
Zero-Coupon Bond
15% Coupon Bond and YTM of 6%
3% Coupon Bond and YTM of 15%
15% Coupon Bond and YTM of 15%

Convexity: a quadratic adjustment (i.e. using the curve's curvature)



Duration is a linear approximation which is not taking into account that the price-yield relationship of a bond is convex (i.e. not linear). Convexity is a desirable return feature.

- Used alone, duration provides a reasonable estimate for small changes in yield, but erroneous estimates for any significant change in yield.
- The duration approximation understates the value of the bond (i.e. underestimate the increase in price when yields fall while overestimating the decrease when yields rise).
- Convexity is a quadratic adjustment to duration which helps provide better estimates.

$$C = \frac{1}{P}\frac{d^2P}{dy^2} = \frac{1}{P\left(1+y\right)^2}\sum_{t=1}^{T}\left(t^2+t\right)\frac{CF_t}{\left(1+y\right)^t} \cong \frac{P_- + P_+ - 2P_0}{P_0\left(\triangle y\right)^2}$$

$$\frac{\triangle P}{P} = -D^*\triangle y + \frac{C}{2}\left(\triangle y\right)^2 \rightarrow P_{y+\triangle y} = P_y \times \left(1 - D^* \times \triangle y + \frac{C}{2}\left(\triangle y\right)^2\right)$$

Example of using duration and convexity together

Calculate the approximate convexity of a 3-year bond with an annual coupon of 3% and a yield to maturity of 3% ($P_0 = 100$), if for a yield change of 0.1% $P_- = 100.283$ and $P_+ = 99.718$.

$$C \cong \frac{P_- + P_+ - 2P_0}{P_0 \left(\triangle y\right)^2} = \frac{100.283 + 99.718 - 2 \times 100}{100 \times (0.001)^2} = 10.877$$

Calculate the expected price of a 3-year bond (with an annual coupon of 3%, a yield to maturity of 3% and a market price of 100) assuming a uniform yield increase of 1%, knowing that the duration is 2.829 and the convexity 10.877.

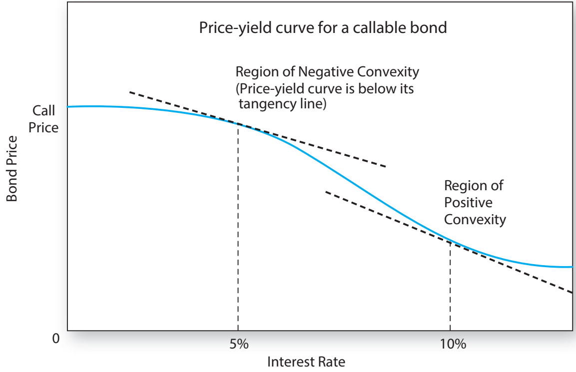
$$\begin{aligned} P_{4.0\%} &= P_{3.0\%} \times \left(1 - D^* \times \triangle y + \frac{C}{2} \left(\triangle y\right)^2\right) \\ &= 100 \times \left(1 - 2.829 \times 0.01 + \frac{10.877}{2} \times (0.01)^2\right) = 97.23 \end{aligned}$$

The same approach can be used to estimate a change in price from a change in spread.

Convexity of a callable bond

As yields fall, a callable bond cannot be worth more than its call price (i.e. price compression).

- In the region of negative convexity $\rightarrow$ interest rate increases result in larger price declines than gains from rate declines of same magnitude, an unattractive asymmetry.
- Practitioners use Effective duration $= -\frac{\triangle P}{P}\frac{1}{\triangle r} = -\frac{P_{-} - P_{+}}{P}\frac{1}{\triangle r}$.



Passive management of fixed-income portfolios

Passive management assumes bond prices are fair, that there no benefit to forecast changes in interest rates (i.e. no use of market timing), and it is more useful to focus on risk management.

Indexing

- Replicate the performance of a given bond index (get the same risk exposure).
- Replicating an equity index is easy as its composition is stable and has few constituents.
- As replicating a bond index is not practical, a stratified sampling approach is used (portfolio weights according to a matrix of asset sub-classes and maturities).

Immunization

- Match the risk exposure of your assets and liabilities in seeking a zero-risk profile (i.e. changes in interest rates do not result in significant gains nor losses).
- Matching duration results in price risk and reinvestment risk to cancel out while the value of assets and liabilities rise (fall) in sync from a fall (rise) in interest rates.
- See tables 16.4 and 16.5 of textbook for examples.

Use forecasts of changes in interest rates: if interest rates declines (increases) are anticipated then increase (decrease) portfolio duration (i.e. use market timing to get abnormal returns).

Arbitrage relative mispricing within the fixed-income market (i.e. buy undervalued bonds and sell overpriced bonds) until the mispricing disappear.

Portfolio rebalancing strategies

- Substitution swap (substitute nearly identical bonds for better return)
- Intermarket spread swap (shift sector in anticipation to changes in spreads)
- Rate anticipation swap (alter portfolio in anticipation of changes in yields)
- Pure yield pickup swap (hold higher yield bonds)
- Tax swap (exploit some tax advantages)

Horizon analysis

- Forecast the yield curve at the end of the investment horizon (i.e. holding period) and alter the composition of the portfolio to maximize anticipated total return.

Chapter 16 of the textbook

Learning Objectives covered

- L01 to L04

Concept checks

- Concept checks 1 to 8 (solutions provided at the end of the chapter).

Exercises

- Suggest solving 16-9, 16-11 and 16-14.

Time to pay	Amount	YTM@T=0	PV	Weight	Duration
1	10,000,000	10%	9,090,909	0.7854	0.7854
5	4,000,000	10%	2,483,685	0.2146	1.0729
Total			11,574,594	1.0000	1.8583

a) Zero-coupon bond of maturity: 1.8583

b) Present value of of obligations: 11,574,594

Present value of asset needed: 11,574,594

Face value of zero-coupon bond: 13,817,414

16-11 Solution

Maturity	Spot@T=0	Spot@T=0	Coupon	Price@T=0	Price@T=1	Gain/loss	Coupon	Return
1	7.00%	9.00%	8.00%	$1,009.35	$1,000.00	-$9.35	$80.00	7.00%
2	8.00%	9.00%	8.00%	$1,000.00	$990.83	-$9.17	$80.00	7.08%
3	9.00%	9.00%	8.00%	$974.69	$982.41	$7.72	$80.00	9.00%

The three-year bond is providing the highest return and is therefore the appropriate choice given the expectation that the yield curve will be flat at 9% by year-end.

16-14 Solution

Portfolio:	1,000,000	Bond	Maturity	YTM	Duration	Weight
Duration:	10	Zero-coupon	5	5%	5.0	0.6875
		Pepetuity	n/a	5%	21.0	0.3125
						1.0000

$(w \times 5) + [(1 - w) \times 21] = 10$, so $w = 11/16 = 0.6875$

Portfolio:	1,000,000	Bond	Maturity	YTM	Duration	Weight
Duration:	9	Zero-coupon	4	5%	4.0	0.7059
		Pepetuity	n/a	5%	21.0	0.2941
						1.0000