UNIVERSITY OF CALGARY
HASKAYNE SCHOOL OF BUSINESS

Investments & Portfolio Management

Investor Preferences and Capital Allocation

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A straightforward approach to portfolio construction consists in deciding first the capital allocation between risk-free and risky assets, and then choosing the composition of the portfolio of risky assets.

But various investors have different willingness and ability to bear ('tolerate') risk. So, a portfolio manager needs **a rational approach to optimize the risk-return profile of a portfolio** of securities by simultaneously taking into account expected returns and the risk tolerance of a given investor. This is achieved using an utility function to rationally account for the risk aversion of the investor. One can then choose the portfolio which delivers the highest utility, and therefore optimize the trade-off between risk and return in achieving the optimal allocation of capital between risky and risk-free assets for a given investor.

A simple approach to devise the composition of the portfolio of risky assets is to mimic a well-diversified index of widely held stocks (like the S&P500). Nowadays, this can be achieved on a very cost efficient basis by investing in an ETF like the SPDR S&P 500.

The concept of risk aversion can be illustrated by the decision of a given individual in response to a simple choice between:

- Get \$50 with certainty, or
- Get either \$100 or noting with equal probability (a certainty equivalent gamble of \$50).

Risk seeker (aka risk loving, $A < 0$)

- A 'risk seeking' individual choosing the gamble might enjoy the uncertainty and be even interested in negative expected returns (e.g. gambling at the casino).

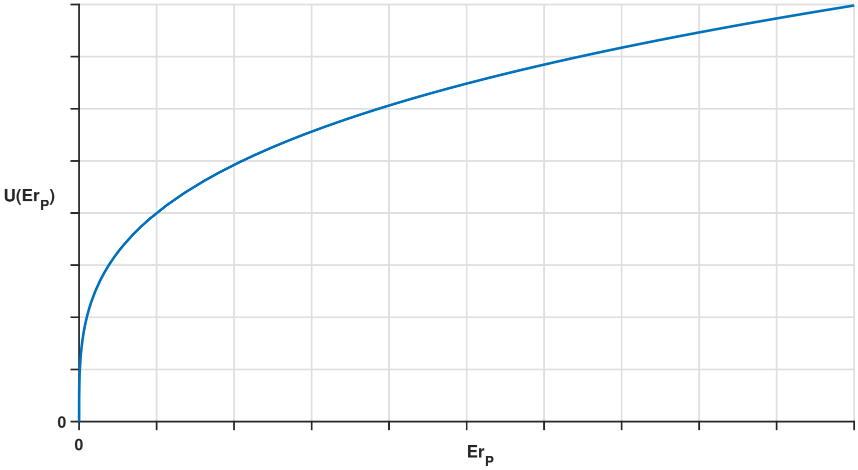
Risk neutral ($A = 0$)

- An individual being indifferent between either options is said to be risk neutral (i.e. only interested in return and not by risk).

Risk averse ($A > 0$, prevalent given historical positive relationship between returns and risk)

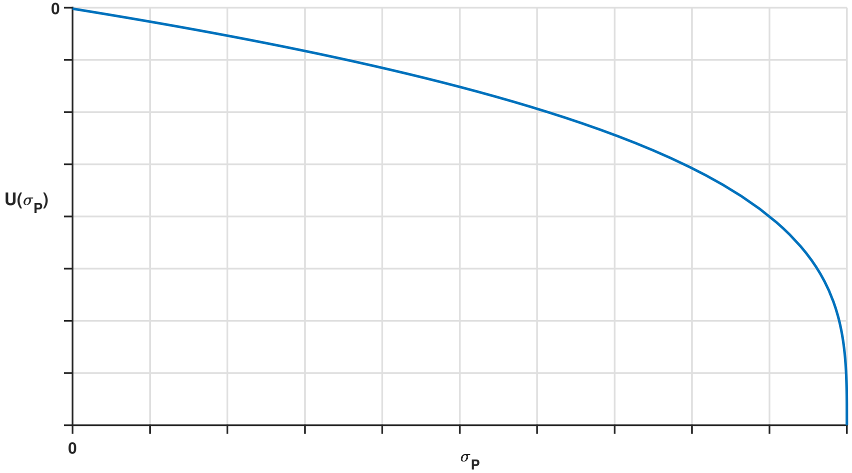
- An individual choosing the guaranteed outcome is said to be risk adverse (i.e. reject the gamble since the associated uncertainty is not being compensated with extra return).

Utility of the expected return



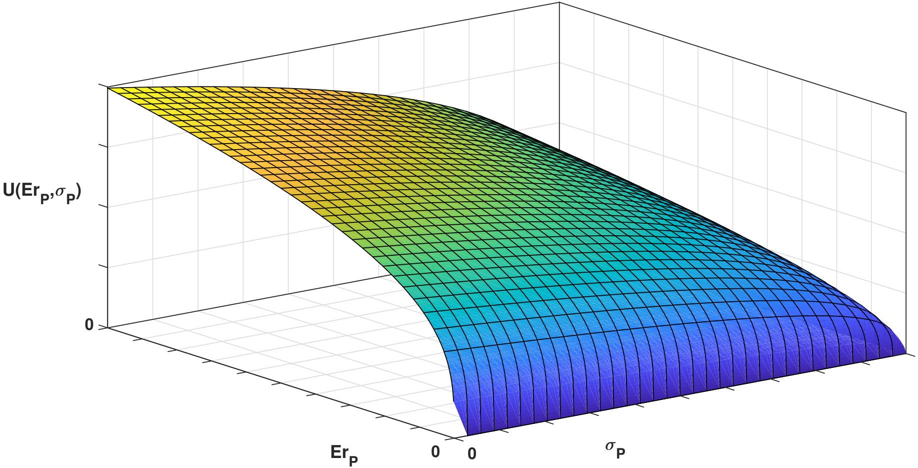
As the expected return increases, its utility increases, but at a declining rate.

Dis-utility of the risk borne (σ of Er_p)

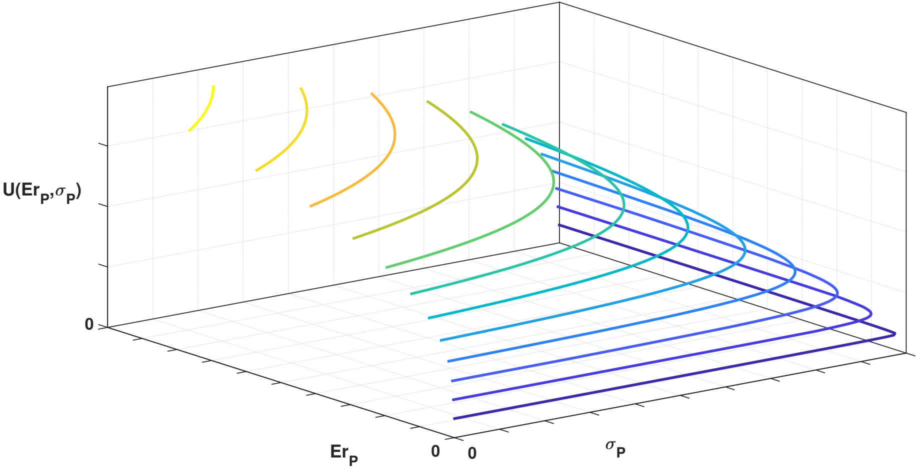


As the standard deviation of the expected return increases, its utility decreases at an increasing rate.

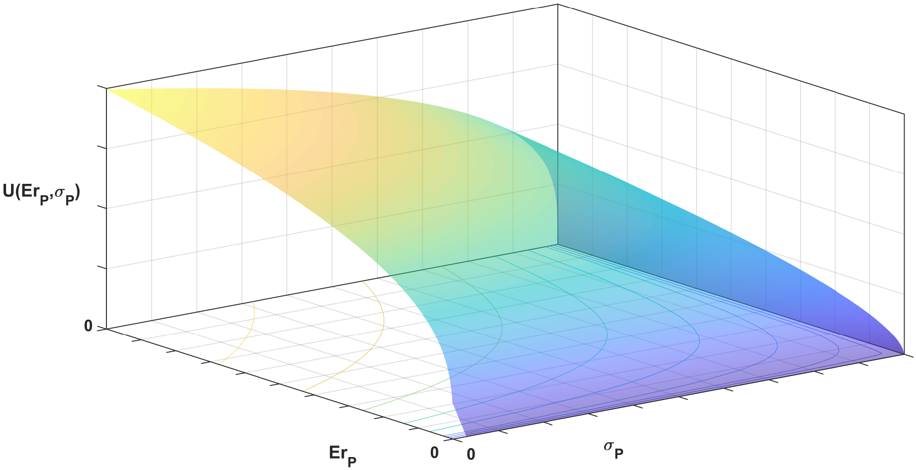
Joint utility function (a surface)



$U\,(Er_p, \sigma_p)$ is the joint utility of Er_p and σ_p

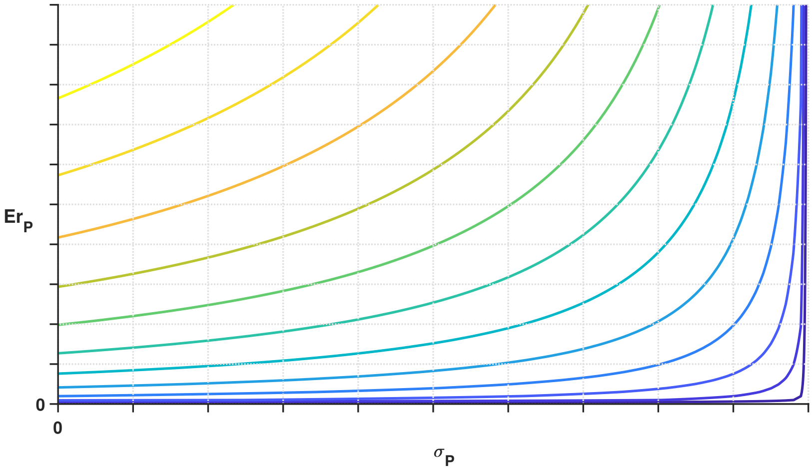


Indifference curve: same utility for various combinations of Er_p and σ_p

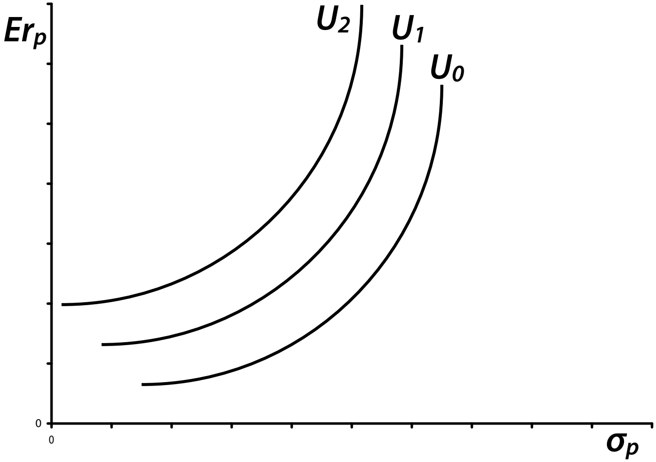


Project the indifference curves on the of Er_p $\sigma_p$plane.

Utility indifference curves for Er_p and σ_p



The indifference curves increase in utility in the direction of the north-west quadrant.



$U_0 < U_1 < U_2$ A portfolio with a higher expected return (more Er_p) and and lower risk (less σ_p) is better.

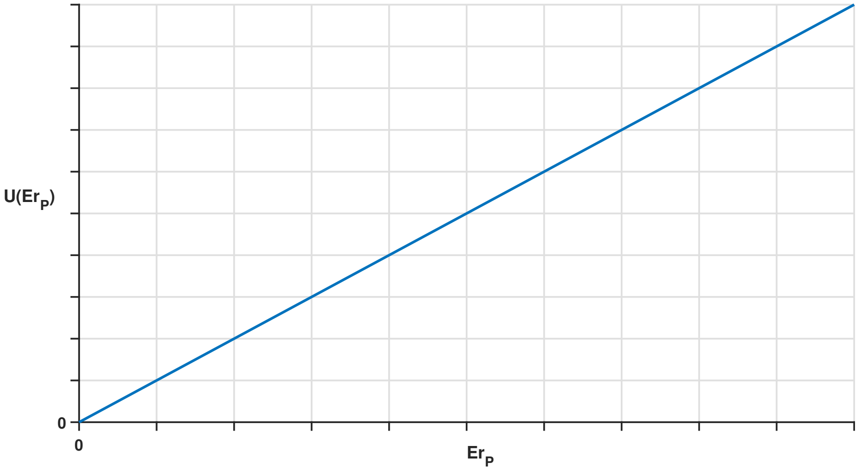
A model of risk aversion using utility theory (many models exist)

$$U_p = F\left(Er_p, \sigma_p, A\right) = Er_p - \frac{1}{2}A\sigma_p^2$$

- A is a coefficient of risk aversion (investor specific, e.g. to be determined using a questionnaire).
- A measures the 'marginal reward that an investor requires to accept additional risk'.
- An investor will choose the portfolio with the highest U_p (according to Er_p, σ_p^2 and A).
- The example below illustrates that the investor with a high risk tolerance ($A = 2$) chooses the high risk portfolio, while the investors with a lower risk tolerance (higher A) choose a lower risk portfolio).
- Studies have found that the representative investor has a risk aversion between 2 and 4.

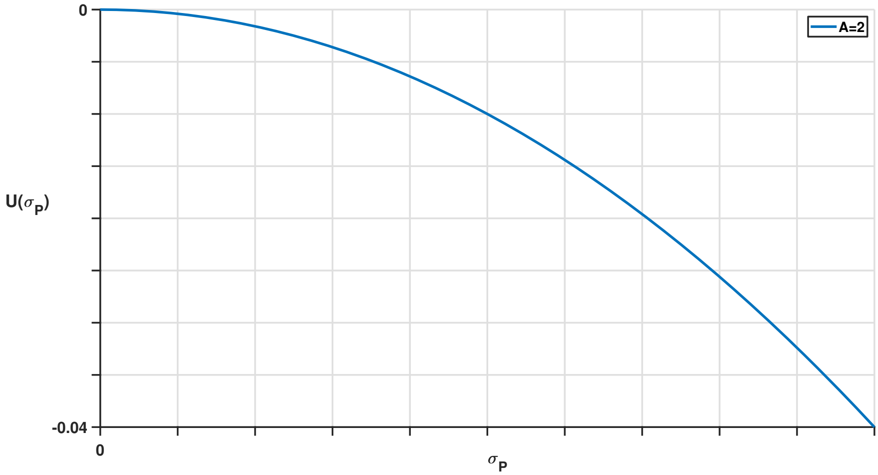
Investor's A	Low risk portfolio $Er_p = .07$ $\sigma_p = .05$	Medium risk portfolio $Er_p = .09$ $\sigma_p = .10$	High risk portfolio $Er_p = .13$ $\sigma_p = .20$
2.0	$.07 - \frac{1}{2} \times 2.0 \times .05^2 = .0675$	$.09 - \frac{1}{2} \times 2.0 \times .10^2 = .0800$	$.13 - \frac{1}{2} \times 2.0 \times .20^2 = \mathbf{.0900}$
3.5	$.07 - \frac{1}{2} \times 3.5 \times .05^2 = .0656$	$.09 - \frac{1}{2} \times 3.5 \times .10^2 = \mathbf{.0725}$	$.13 - \frac{1}{2} \times 3.5 \times .20^2 = .0600$
5.0	$.07 - \frac{1}{2} \times 5.0 \times .05^2 = .0638$	$.09 - \frac{1}{2} \times 5.0 \times .10^2 = \mathbf{.0650}$	$.13 - \frac{1}{2} \times 5.0 \times .20^2 = .0300$

Utility of the expected return per $U_p = Er_p - \frac{1}{2}A\sigma_p^2$



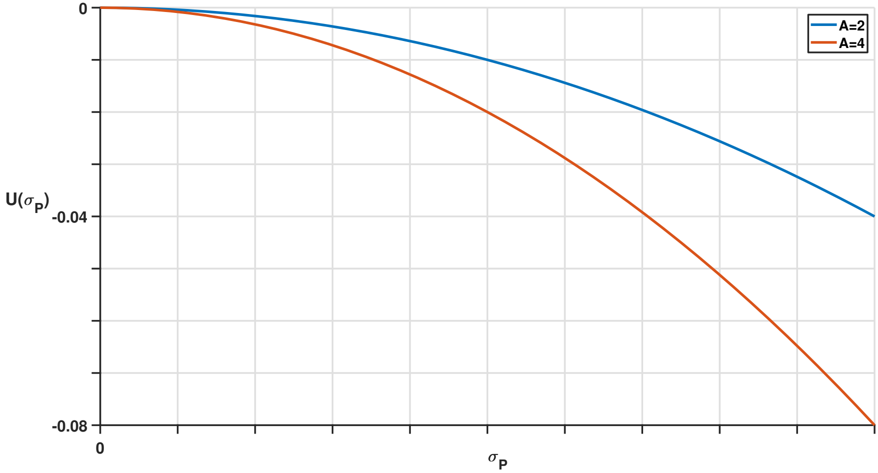
As the expected return increases, its utility increases, at a linear rate.

Dis-utility of the risk borne per $U_p = Er_p - \frac{1}{2}A\sigma_p^2$



As σ_p increases, its utility decreases at an increasing rate as per risk aversion coefficient.

Dis-utility of the risk borne per $U_p = Er_p - \frac{1}{2}A\sigma_p^2$



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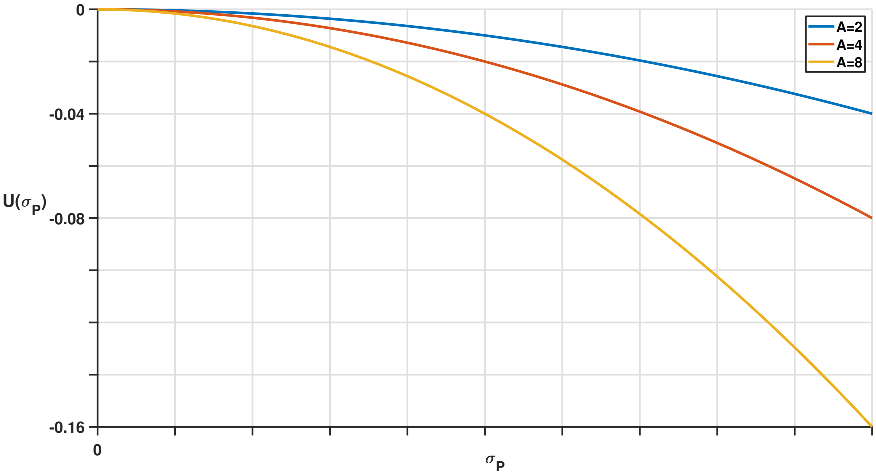

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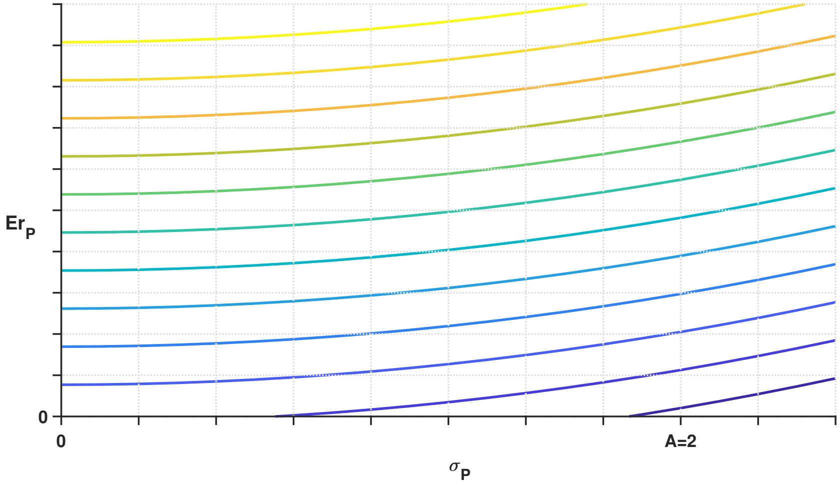
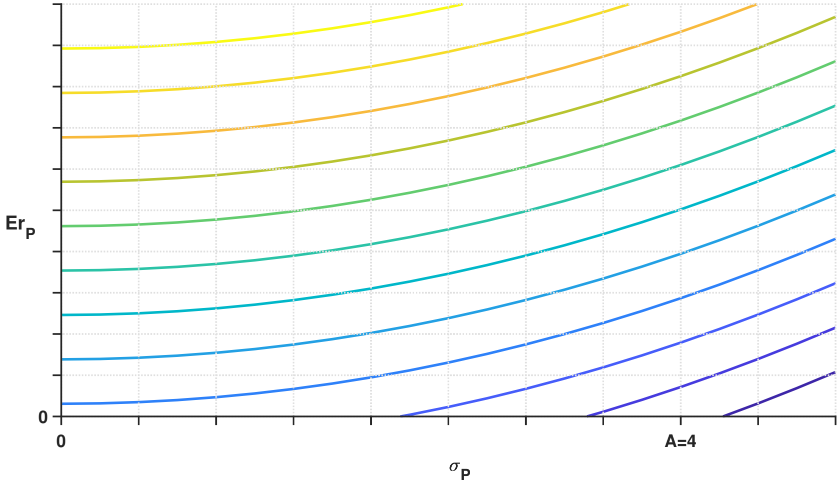


The indifference curves increase in utility in the direction of the north-west quadrant.



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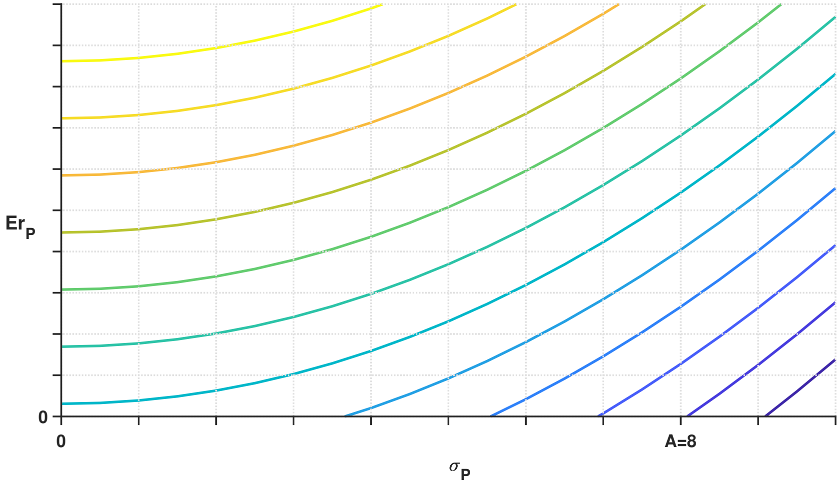
Utility indifference curves per $U_p = Er_p - \frac{1}{2}A\sigma_p^2$



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Consider the existence of 'risk-free securities'...

- A central government can issue 'default-free' fixed-income securities in domestic currency since a central government by virtue of issuing its own currency can print more of it to repay its fixed-income debt obligations and therefore avoid defaulting.
- But 'printing money' debases the currency and induces much higher inflation, depressing the real return earned by investors. So 'default-free' is not 'risk-free' if the issuer can expropriate via choosing to induce much higher inflation than investors expected.
- Only a central government of an advanced economy having too much to loose from excessive inflation coupled with an independent central bank with a stable inflation track record can reasonably be expected not to take undue advantage of fixed-income investors.
- Of note, some central governments have issued for some time securities with some form of inflation protection (e.g. TIPS and RRB).

For the purpose of this course we will assume the existence of a risk-free asset.

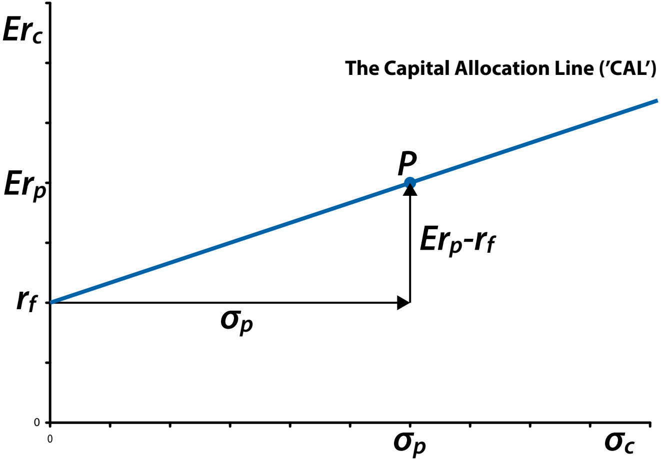
Let's consider a portfolio c combining risk-free (i.e. $\sigma_f = 0$) and risky securities. A proportion y of total value is allocated to a portfolio p of risky securities.

$$Er_c = (1-y)\, r_f + yEr_p = r_f + y\,(Er_p - r_f)$$

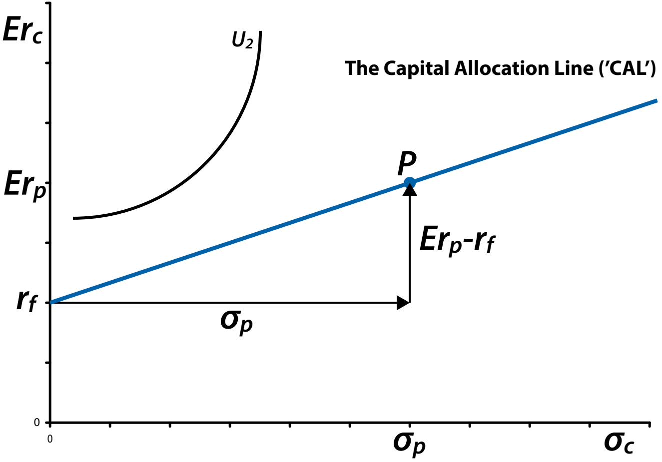
$$\sigma_c^2 = (1-y)^2\sigma_f^2 + y^2\sigma_p^2 + 2\rho_{fp}(1-y)y\sigma_f\sigma_p = (1-y)^2 0 + y^2\sigma_p^2 + 20(1-y)y0\sigma_p = y^2\sigma_p^2$$

$$\sigma_c = y\sigma_p \rightarrow y = \frac{\sigma_c}{\sigma_p} \rightarrow Er_c = r_f + \frac{(Er_p - r_f)}{\sigma_p}\sigma_c$$

The expected return for portfolio c is linear in the risk of the portfolio p of risky securities with a compensation for risk of $\frac{(Er_p - r_f)}{\sigma_p}$ (i.e. a risk premium) and an intercept of r_f.



Some commonality with CAPM, but this only illustrates an asset allocation between risk-free and risky securities.



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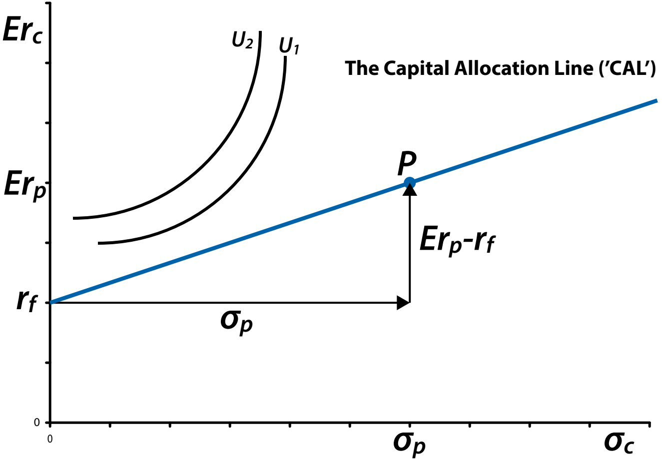

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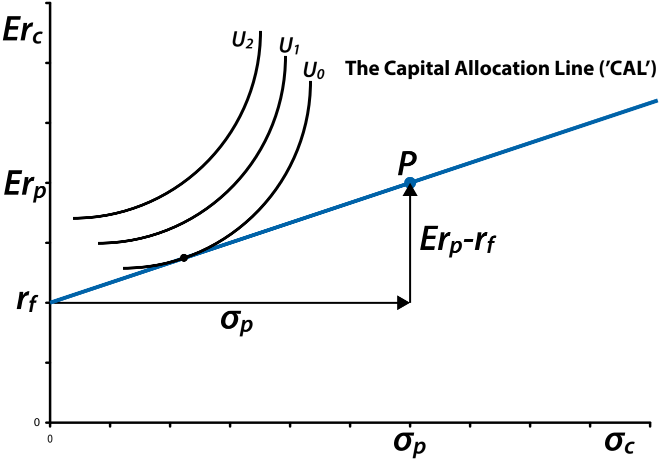

Some commonality with CAPM, but this only illustrates an asset allocation between risk-free and risky securities.

Let's find what would be the optimal asset allocation y^* for a given risk aversion A, by considering which utility function should be maximized and then finding its maximum by using its first derivative with respect to y to equal 0.

$$MAX_y \; U_c = Er_c - \frac{1}{2} A\sigma_c^2 = r_f + y\,(Er_p - r_f) - \frac{1}{2} A\, y^2 \sigma_p^2$$

$$(Er_p - r_f) - A\, y^* \sigma_p^2 = 0 \rightarrow y^* = \frac{Er_p - r_f}{A\, \sigma_p^2}$$

This quantitative approach can be used to specify the optimal asset mix (e.g. 40:60) as the split between the risk free asset (e.g. ETF of short-term government bonds or a money market fund) and the risk asset (e.g. ETF of widely held stocks) once the inputs have been quantified.

Applications in chapter 6 of the textbook

Concept checks

- Suggest to do concept checks 1 to 8 (solutions provided at the end of the chapter).

Exercises

- Suggest 6-4, 6-10, 6-11, and 6-12.
- Solutions follow next slides, and Excel solution file in available in D2L.

	Probability	Worth	Rf	Risk prem.	Rate of Ret.	PV
Scenario 1	0.5	70,000	0.06	0.08	0.14	118,421
Scenario 2	0.5	200,000	0.06	0.12	0.18	114,407
Certainty eq.		135,000				

a. Required rate of return = Risk-free rate + Risk premium = 0.06 + 0.08 = 0.14
   Present value = cash-flow / discount rate = 135,000/1.14 = 118,421

b. Rate of return = (CF1 / CF0) - 1 = 0.14

c. Required rate of return = Risk-free rate + Risk premium = 0.06 + 0.12 = 0.18
   Present value = cash-flow / discount rate = 135,000/1.18 = 114,407

d. For a given expected cash flow, portfolios that command greater risk premiums
   must sell at lower prices. The extra discount from expected value is a penalty for risk.

6-10, 6-11 and 6-12 Solution

T-Bill rate	5%
Risk Premium	8%
Index return	13%
SD return	20%

W bill	W index	ER	VAR	U (A=2)	U (A=3)
0.0	1.0	13.0%	4.0%	**0.0900**	0.0700
0.2	0.8	11.4%	2.6%	0.0884	0.0756
0.4	0.6	9.8%	1.4%	0.0836	**0.0764**
0.6	0.4	8.2%	0.6%	0.0756	0.0724
0.8	0.2	6.6%	0.2%	0.0644	0.0636
1.0	0.0	5.0%	0.0%	0.0500	0.0500

Investors with A = 2 prefer a portfolio that is invested 100% in the market index to any of the other portfolios in the table.
The more risk averse investors prefer the portfolio that is invested 60% in the market, rather than the 100% market weight preferred by investors with A = 2.