UNIVERSITY OF CALGARY
HASKAYNE SCHOOL OF BUSINESS

Investments & Portfolio Management

Risk and Return

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Determinants of the Level of Interest Rates

Interest rates determine the expected returns when investing in low risk fixed income securities. In turn, when investing in riskier securities like corporate bonds or equity, higher returns are expected to compensate for the additional risks borne.

Interest rates are influenced by the interaction between the supply and demand of funds, the expected rate of inflation, taxation and by monetary policies as decided and implemented by the central bank as well as the public sector net demand for funds.

- Intuitively, the higher the interest rates the more incentive is provided to households to save. However, counterintuitively, low interest rates incentivize households to save more to achieve the same level of income and standard of living at retirement.
- Intuitively, the lower the interest rate the more incentive is provided to firms to borrow (as a lower cost of capital increases NPVs). However, counterintuitively, firms will borrow to increase capital expenditures at times of good economic prospects which typically coincide with higher than normal interest rates and vice-versa during economic slowdown.

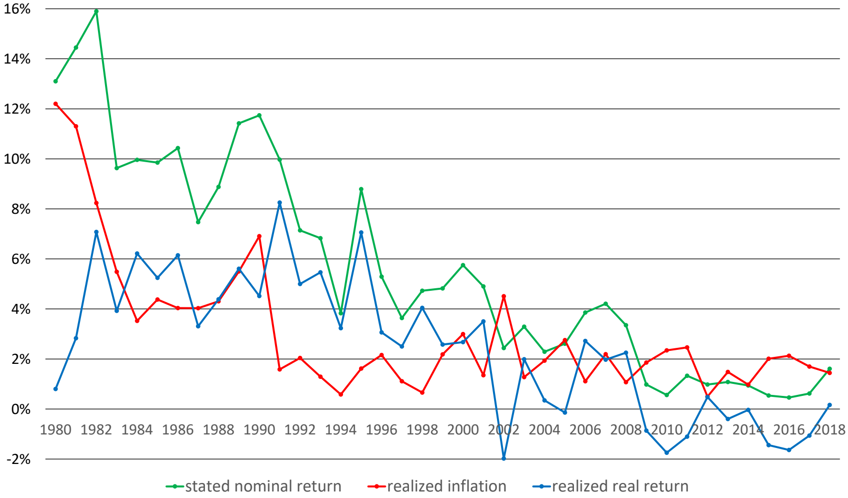
For households, saving is moving consumption from the present to the future (vice versa for borrowing, e.g. student loans). So, the appropriate expected returns shall be calculated in units of future consumption, which can be proxied by after-tax real returns (i.e. how much actual consumption you expect to receive in the future for each unit of consumption foregone today).

The Fisher hypothesis suggests $r_{nominal} = r_{real} + Ei$, that the real rate being constant, changes in nominal rate are best explained by changes in expected inflation. However, households often focus on nominal pre-tax returns which is underestimating the negative joint effects of inflation and taxation which drag down actual future consumption from savings (especially if inflation and taxation turn out to be higher than expected).

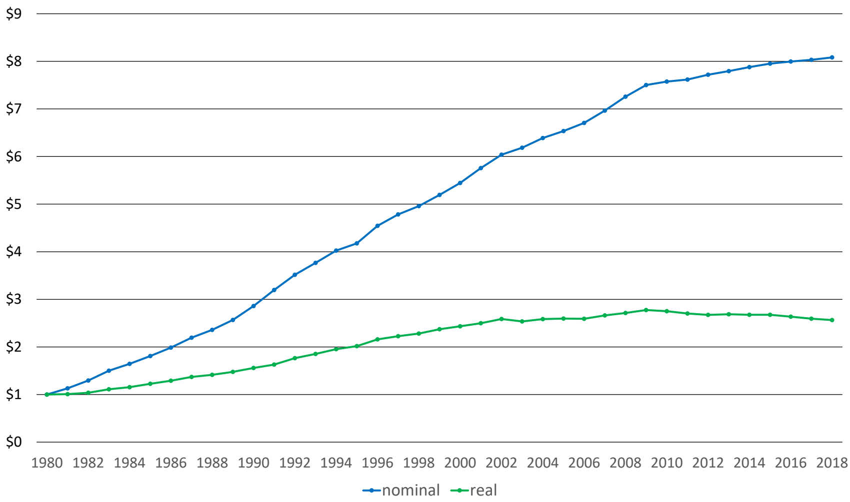
$$1 + r_{nominal} = (1 + r_{real})(1 + i) \rightarrow r_{real} = \frac{r_{nominal} - i}{1 + i} \rightarrow r_{real} \simeq r_{nominal} - i$$

$$r_{after-tax\ real} \simeq r_{nominal} \times (1 - t) - i = 1.6\% \times (1 - 0.4) - 1.9\% = -0.94\%$$

1-year CAD TBill Realized Returns



1-year CAD TBill Compounded Growth of $1



Total return rate for zero-coupon government securities (simple interest, not annualized)

$$r_{f,T} = \frac{F}{P_0} - 1$$

Effective annual rate ('EAR'), a compound interest

$$EAR = [1 + r_{f,T}]^{\frac{1}{T}} - 1$$

Annual percentage rate ('APR'), by convention for $T < 1$ and $n = \frac{1}{T}$ periods per year

$$APR = n \times r_{f,T} \rightarrow EAR = [1 + T \times APR]^{\frac{1}{T}} - 1 \rightarrow APR = \frac{(1 + EAR)^{T} - 1}{T}$$

Continuous compounding (as $T \rightarrow 0$)

$$EAR = e^{r_{cc}} \rightarrow r_{cc} = ln\,(1 + EAR)$$

$$HPR = \frac{P_T - P_0 + cash\ received}{P_0} \rightarrow annualized\ HPR = (1 + HPR)^{\frac{1}{T}} - 1$$

Expected return and standard deviation according to a scenario (Excel spreadsheet file 5.1)

$$Er = \sum_s p_s r_s \qquad \sigma^2 = \sum_s p_s \left[r_s - Er\right]^2 \qquad \sigma = \sqrt{p_s \left[r_s - Er\right]^2}$$

Realized returns and standard deviation (Excel spreadsheet file 5.2)

$$\overline{r} = \frac{1}{n} \sum_{s=1}^{n} r_s \qquad \hat{\sigma}^2 = \frac{1}{n-1} \sum_s \left[r_s - \overline{r}\right] \qquad \hat{\sigma} = \sqrt{\frac{1}{n-1} \sum_s \left[r_s - \overline{r}\right]}$$

The Sharpe ratio (a reward-to-volatility metric)

$$Sharpe\ ratio = \frac{risk\ premium}{\sigma\ of\ excess\ return}$$

Arithmetic mean of realized returns (also noted u):

$$\overline{R_a} = \frac{R_1 + R_2 + \ldots + R_T}{T} = \frac{1}{T}\sum_{i=1}^{T} R_i$$

- 'typical' return for a single period over the holding period

Geometric mean of realized return:

$$\overline{R_g} = [(1+R_1)\times(1+R_2)\times\ldots\times(1+R_T)]^{\frac{1}{T}} - 1 = \left[\prod_{i=1}^{T}(1+R_i)\right]^{\frac{1}{T}} - 1$$

- 'earned' return over the holding period as if a single period

$\overline{R_a} = \overline{R_g}$ only if riskless (R is constant), if not (assuming normality of R):

$$\overline{R_a} > \overline{R_g} \leftrightarrow \overline{R_a} - \sigma^2/2 \simeq \overline{R_g}$$

$\sigma^2/2$ is known as the 'volatility drag' (higher actually since non-normality).

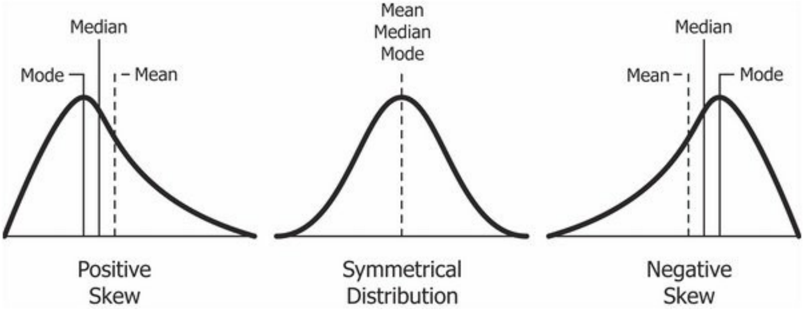
A normal distribution is fully characterized with two parameters: its mean and standard deviation. It is often assumed that financial returns are normally distributed (it makes the math easy). If it was the case the third moment and the fourth moment of realized retuns (i.e. their skewness and kurtosis) would equal 0. However, the evidence suggests this not the case. Financial returns often exhibit positive skewness and are typically leuptokurtic (i.e. fat-tailed).

The value at risk ('VAR'): a 1% VaR indicates that 1% of returns will lie below that value.

$$VaR@1\% = mean - 2.33\sigma \qquad (assuming\ normal\ returns)$$

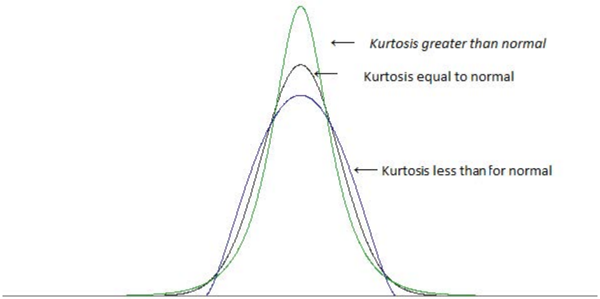
The expeted shortfall ('ES'): how much is lost on average with all 1% worst returns.

The lower partial standard deviation ('LPSD') is the standard deviation of negative excess returns and the Sorentino ratio is the ratio of average excess returns to LPSD.



Source: Jain (2018)

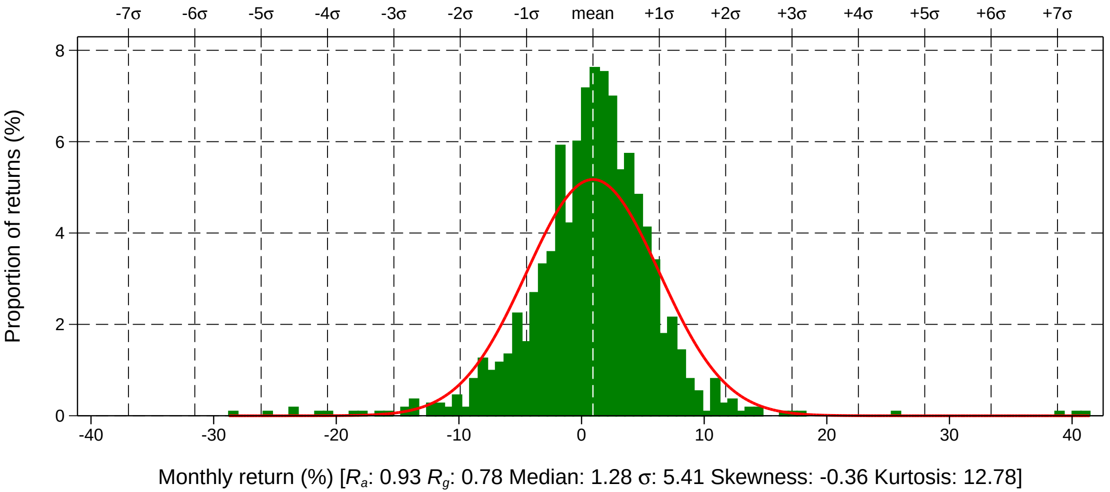
- Investors like the third moment of returns in the same way they like the first moment, the mean return (i.e. positive skewness is 'good' while negative skewness is 'bad').
- Skewness between -0.5 to 0.5 is 'small' (i.e. still afairly symmetrical distribution).



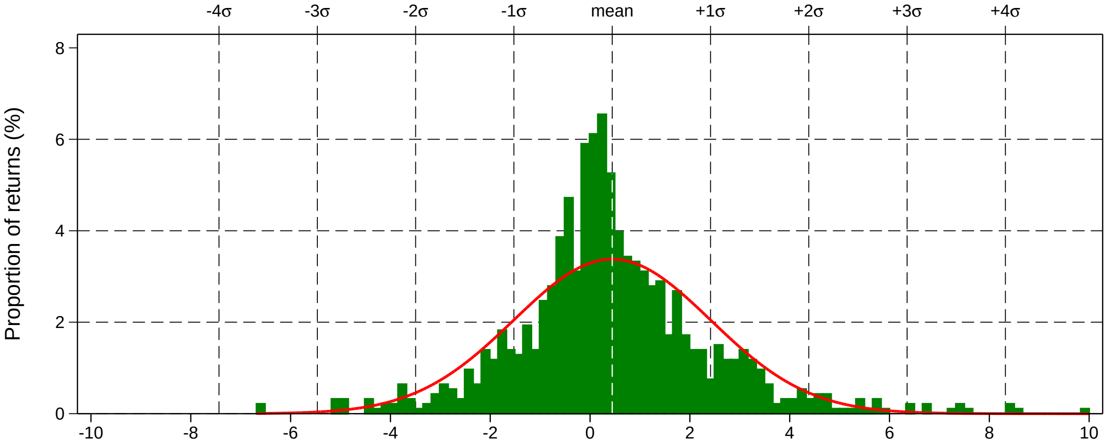
Source: Monica (2015)

- Investors like the fourth moment of returns in a somewhat similar way they like the second moment, the variance (i.e. positive kurtosis is 'good' for positive returns while negative kurtosis is 'bad' for negative returns).
- Kurtosis for a normal distribution is 3, but often kurtosis is reported with 3 subtracted.

The distribution of S&P500 monthly returns (1926 to 2018)



The distribution of US 10Y TBond monthly returns (1941 to 2018)

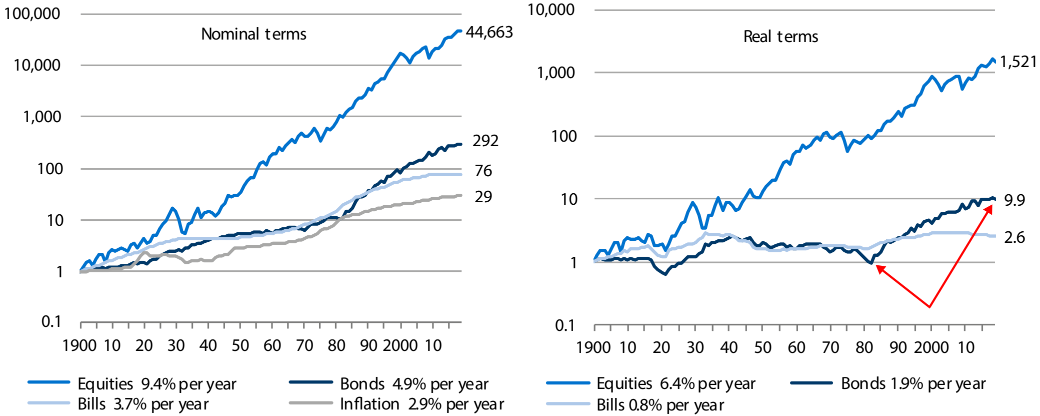


Monthly return (%) [R_a: 0.444 R_g: 0.425 Median: 0.255 σ: 1.97 Skewness: 0.55 Kurtosis: 5.42]

	Return (%)	Risk (%)
Large company stocks	10.2	19.8
Small company stocks	12.1	31.7
Long-term corporate bonds	6.1	8.3
Long-term government bonds	5.5	9.9
Treasury bills	3.4	3.1
Inflation	2.9	4.0

Source: SBBI Yearbook (2018), returns are annualized geometric mean returns and risk is annualized monthly std. deviations.

Using historical mean returns to represent expected returns is a useful but often ill-advised assumption commonly made (i.e. think $1 + ER = (1 + r_f) \times (1 + Ei) \times (1 + ERP)$ versus R).



Source: Dimson, Marsh and Staunton, Triumph of the Optimists (2002), and Global Investment Returns Yearbook, Credit Suisse (2019)

The real returns of bonds prior to 1980 were low because of high inflation, but high 1980 onward, as both the interest rates and inflation went from very high to very low levels.

Long term returns are a consequence of the compounding process of short term returns. Compounding normal distributions (i.e. $(1+r_1)(1+r_2)\ldots$) does not result in a normal distribution but a lognormal distribution (i.e. it is the log of final wealth W_t which is normally distributed).

$$EW_T = W_0 e^{Er_{cc}T} \to \sigma(r_{cc}T) = \sigma(r_{cc})\sqrt{T}$$

The expected return after T is $Er_{cc}T$, which increases at a faster rate than the risk $\sigma(r_{cc})\sqrt{T}$. This could be interpreted as if long term returns are being less risky than short term returns. However, this is a fallacy since while the probability of losses decrease with time, the magnitude of potential losses increase (see example 5.11).

One way to think about the root cause for the lack of symmetry of long term returns is that the positive returns (upside potential) are unlimited while the negative returns are not because of limited liability.

Concept checks

- Suggest to do concept checks 1 to 7 (solutions provided at the end of the chapter).

Exercises

- Suggest 5-6 and 5-8.
- Solutions follow next slides, and Excel solution file in available in D2L.
- If you have time, suggest doing 5-12.

5-6 Solution

a. The inflation protected investment is safer because it includes a purchasing power guarantee, while providing at least a 1.5% return.

b. If the expected inflation is less than 3.5%, then the 1-year GIC provides a higher real return than the 1-year RRB (and vice-versa).

c. If the expected inflation is 3%, the expacted GIG real return is 2%, 0.5% higher than the 1-year RRB, making it the best investment **if** the realized inflation over the next year is in fact less than 3.5%. The 'better investment' depends on the investor atitute toward risk versus return. Maybe the 'best investment' would be to diversity and invest half of the funds in each security.

d. Not exactly, as investors assuming an inflation risk likely require to be compensated for bearing that risk. Part of the difference between the risk-free nominal rate of 5% and the real risk-free rate of 1.5% would be made by that risk premium for bearing purchasing power/inflation risk. So logically, the expected inflation is likely slightly smaller than 3.5%.

5-8 Solution

Coupon	8%
Maturity	30
Face value	100
Price	100

State of Economy	Probability	YTM	Price	Gain	Interest	HPR	(R - ER)^2
Boom	20%	11.00%	74.05	-25.95	8.00	-17.95%	0.0597488
Normal	50%	8.00%	100.00	0.00	8.00	8.00%	0.0002270
Recession	30%	7.00%	112.28	12.28	8.00	20.28%	0.0190011
	100%						

Expected Return	3.44%	wrong
Expected Return	6.49%	OK
Expected Return	6.49%	OK
Variance	0.0254	wrong
Variance	0.0178	OK
Standard Deviation	13.33%	