UNIVERSITY OF CALGARY
HASKAYNE SCHOOL OF BUSINESS

Investments & Portfolio Management

Bond Valuation and Yields

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Bonds and bond-like Instruments (typically with maturities more than a year)

- Government Bonds (issued by a state like Government of Canada)
- Inflation-Protected Bonds (e.g. Real Return Bonds issued by Government of Canada).
- Corporate Bond (issued by a corporate, often with a call provision for issuer to pay early)
- Convertible Bonds (issued by a corporate and convertible into equity of said corporate)
- Mortgage-backed security ('MBS', issued by securitized pools of mortgages)
- Asset-backed Security ('ABS', issued by a securitization vehicle)

Interest, aka the coupon rate, is typically paid semi-annually.

The riskiness (i.e. probability of default) of a given fixed-income security is quantified by a credit rating (e.g. from AAA to D) from a credit rating agency (e.g. Moody's).

Bond terminology (some of it...)

Par value: stated value of the bond and principal at maturity ('F')

Coupon: periodic interest payments made by the issuer

Maturity date: date of the last cash flows (e.g. coupon and principal)

Strip: single cash flow at maturity (no periodic payments of any kind)

Yield to maturity: yield implied by the market price of a bond ('YTM')

At premium (above par): market price > par value (coupon > YTM)

At discount (below par): market price < par value (coupon < YTM)

Volatility: how price sensitive a given bond is to changes in interest rates

Holding period return: return over a period at mark-to-market prices, inclusive of net interim cash flows received

Bullet bond (typical of government bonds and 'high quality' corporate issuers)

- For most bonds the principal is repaid at maturity. Such bonds are called 'bullet bonds'.

Fully or partially amortized bond (mitigates credit risk but increases reinvestment risk)

- For some bonds the principal is repaid periodically alongside the interest.
- If the principal is repaid equally in full by the maturity date, it is a fully amortized bond.
- It is a partially amortized bond if a balloon payment retires the remainder at maturity.

Sinking fund arrangement (mitigates credit risk but increases reinvestment risk)

- The issuer pays the bond trustee a certain percentage of the bond outstanding principal on an annual basis, which cash is used to redeem bonds on a pro-rata basis or by lottery.

Flow through (cash flow logic, but reinvestment risk negatively correlated with interest rates)

- For residential mortgage backed securities (RMBS), any payment of principal from the underlying mortgages is passed-through to investors, who are therefore exposed for such type of securities to a pre-payment risk which is negatively correlated with interest rates.

Call provisions (aka ‘redemption provisions’)

- A corporate bond which embeds an option allowing the issuer to repurchase the bond at a given ‘call price’ prior to maturity is a ‘callable bond’.
- This provides the issuer flexibility to act opportunistically to use unforeseen extra cash to reduce its indebtedness or to refinance at a lower cost when interest rates fall.
- This flexibility is valuable and issuers have to compensate investors accordingly by offering a slightly higher interest rate than would be the case otherwise.
- The call price usually slightly exceed the par value while investors are typically protected during an initial period during which the call option cannot be exercised by the issuer.

Convertible bond

- The bondholder has the right but not the obligation to convert into a certain number of common shares of the issuer (i.e. a convertible bond is a bond plus an equity call option).
- The conversion feature benefiting the bondholder (i.e. of value), the investor receive a lower coupon that would otherwise be the case (but still higher than the dividend yield).

Alternative coupon structures for corporate bonds

Floating rate notes (valuation is less sensitive to changes in interest rates)

- A bond which pays a variable interest rate is called a 'floating rate note' (FRN).
- The interest rate is a money market rate (reset periodically), plus a fixed spread (e.g. 3-month USD LIBOR + 0.2%). Interest payment matched to periodicity (e.g. quarterly).
- The issuer is typically a financial entity in need of floating rate debt (to match assets and liabilities). Investors can enter into an interest rate swap to get a fixed interest rate.

Set-up coupon

- The interest rate (i.e. the coupon), either fixed or floating, is scheduled to increase at pre-specified dates throughout the life of the bond.
- A set-up feature increases the likelihood that the bond would be called in a stable or decreasing interest rate environment, but offers interest rate risk protection to investors in an increasing interest rate environment or a worst than expected credit risk.

Credit-linked coupon

- The interest rate (i.e. the coupon), either fixed or floating, is increased or decreased according to a credit risk metric (e.g. add/subtract 0.5% per credit rating level below/above the initial credit rating of the issuer at the time of issuance).
- Such credit-linked feature protects investors if the future credit standing of the issuer is somewhat uncertain (while allowing the issuer to raise the funds it requires and to benefit from a lower coupon rate if its credit standing improves).
- But an issuer with substantial such debt might enter a 'death spiral' as increased coupon payments contribute to further deterioration of free cash flows and credit standing.

Payment-in-Kind coupon (PIK)

- A PIK feature allows the issuer to pay interest to bondholders under the form of an additional quantity of the bond (or even with common shares).
- This could be beneficial to both the issuer and investors if the cash flow of the issuer is uncertain, but will likely require the issuer to offer a high yield to prospective investors.

Deferred coupon (aka a split coupon bond)

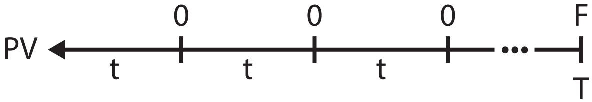
- After an initial period during which the bond pays nothing (e.g. a few years), the interest rate paid is higher than usual for the remainder of the life of the bond.
- Useful for structured finance (e.g. project finance) for the initial ramp-up period during which cash flows from operations are expected to be insufficient to service debt.

Index-linked bonds (aka linkers)

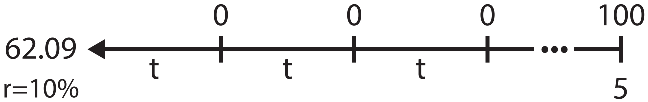
- Coupon and/or principal is linked to an index
  - e.g. if linked to CPI you get an inflation-linked bond which provides investors with a return which is completely or partially free from inflation risk.
- An increase in the index results in an corresponding increase in the coupon and/or the principal, and vice-versa (but such linkage can be structured in many different ways).

Valuation of a 'pure discount bond' (strip)

$$PV = \frac{F}{(1+r)^T}$$

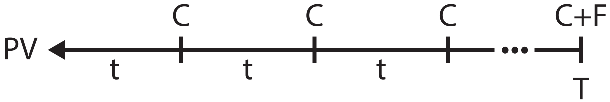


$$PV = \frac{100}{(1+0.1)^5} = 62.09$$

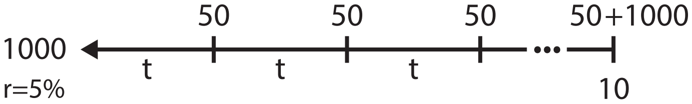


Valuation of a bond (regular paying)

$$PV = C\left[\frac{1}{r} - \frac{1}{r\left(1+r\right)^{T}}\right] + \frac{F}{\left(1+r\right)^{T}}$$



$$PV = 50\left[\frac{1}{0.05} - \frac{1}{0.05\left(1+0.05\right)^{10}}\right] + \frac{1000}{\left(1+0.05\right)^{10}} = 386.09 + 613.91 = 1{,}000$$



Quoted bond prices and accrued interest

Quoted bond prices (aka 'flat prices' or 'clean prices') do not include any interest accruing between coupon payment dates.

Actual price to be paid (aka the 'dirty price' or the 'full price') is the quoted price plus any accrued interest.

$$Accrued\ Interest = \frac{Annual\ coupon\ payment}{2} \times \frac{Days\ since\ last\ coupon\ payment}{Days\ separating\ coupon\ payment}$$

See Excel spreadsheet 14.1 of textbook.

In the previous slides, one practical issue is which r to use? How to find it? In most major markets there is a market for zero-coupon bonds of the sovereign and it is possible to extract the 'risk-free' yield to maturity for many maturities (n) since current prices (P) and face values (FV) are known:

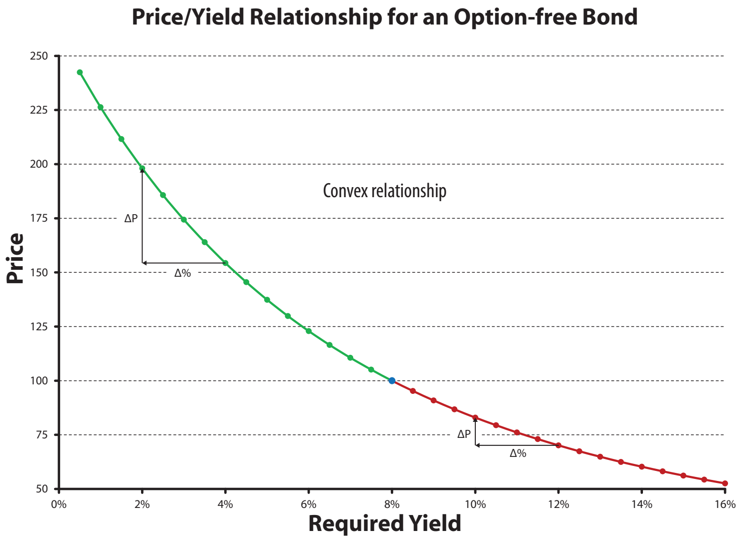
$$YTM_n = \left(\frac{FV}{P}\right)^{\frac{1}{n}} - 1$$

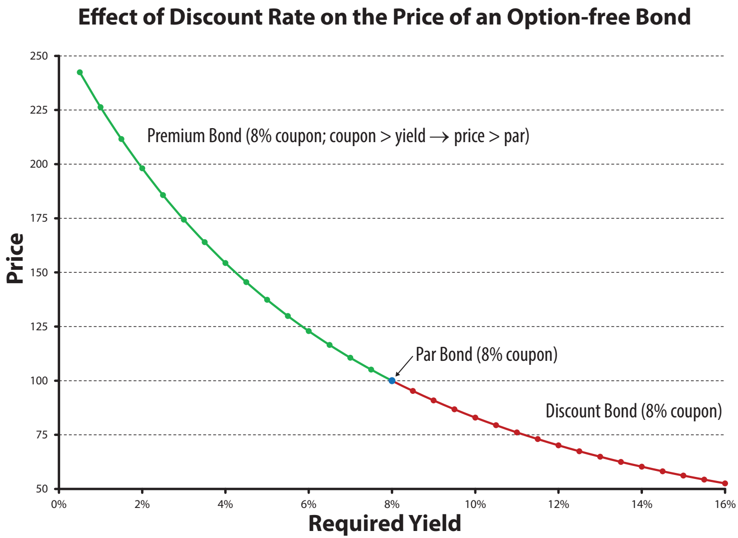
Assuming that you want to value a sovereign bond, one can sum up the present value each cash flow calculated using these YTM_n.

Another possibility is to extract r of very similar bonds using Excel and use an average r.

Finally, for a given type of bond it is possible to estimate its historic risk premium above each past YTM_n and discount cash flows using current YTM_n plus historical average risk premium.

The inverse yield-price relationship is convex (i.e. 'convexity')




Effect of Discount Rate on the Price of an Option-free Bond
Price
250
225
200
175
150
125
100
75
50
Premium Bond (8% coupon; coupon > yield ⟶ price > par)
Par Bond (8% coupon)
Discount Bond (8% coupon)
0%
2%
4%
6%
8%
10%
12%
14%
16%
Required Yield

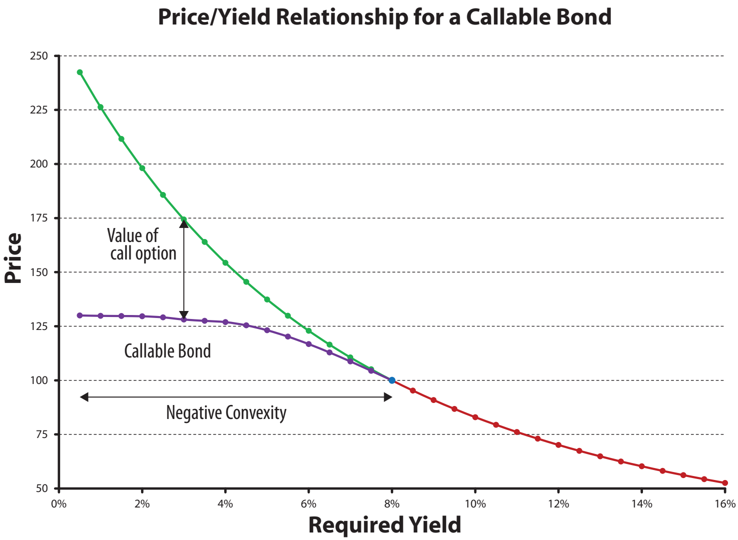
Yield to Call (and yield to put)

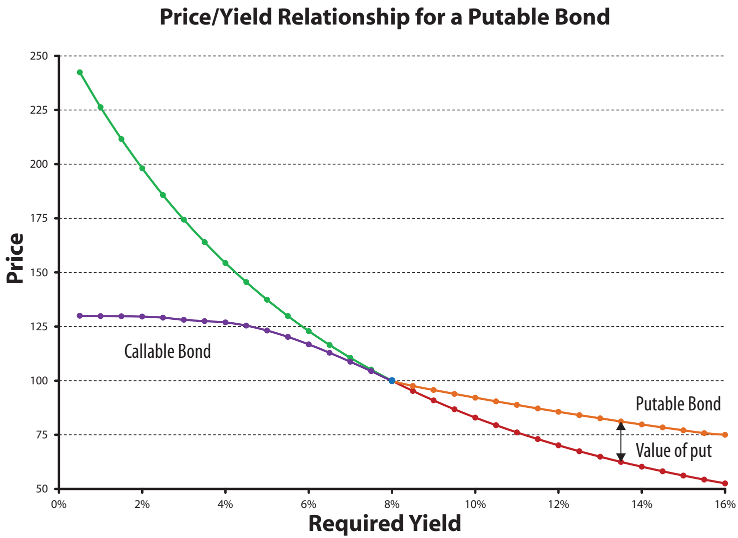
The yield to maturity concept assumes that the bondholder hold the bond until maturity.

However, as soon as the bond embeds an option like a call to the benefit of the issuer (or a put to the benefit of the bondholder), there is a possibility that the bond will be retired prior to maturity. Such probability does not square well with the key assumption of the yield to maturity concept and explain why the yield to call (or yield to put) is a useful alternate yield concept to use for bonds embedding such option (and even more so if it is likely that the option will be triggered).

The above is illustrated on the next slides, and by example 14.5 of the textbook as well as by concept check 14.4.

The yield-price relationship for a callable bond




Price/Yield Relationship for a Putable Bond
250
225
200
175
150
125
100
75
50
Price
Callable Bond
Putable Bond
Value of put
0%
2%
4%
6%
8%
10%
12%
14%
16%
Required Yield

Sources of realized returns on fixed income securities

Investing in a fixed income security provides for three sources of return.

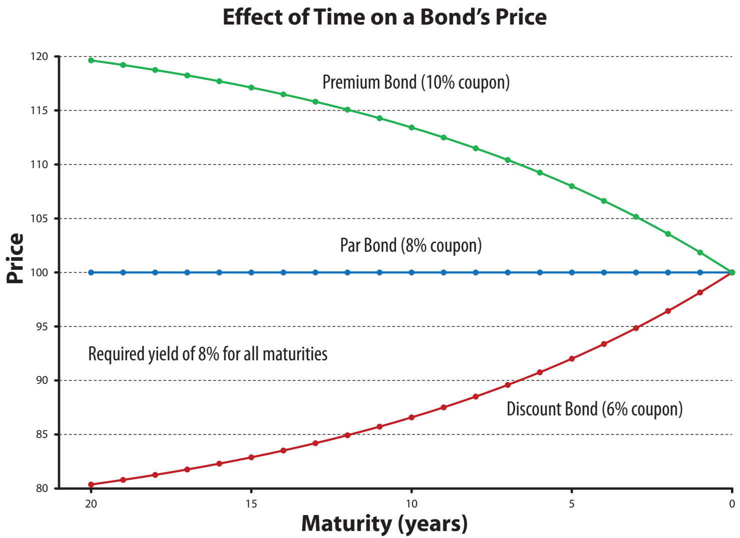
- Receipt of the agreed coupon and principal payments on the scheduled dates;
- Returns from reinvesting the coupon payments received;
- Any capital gain (or loss) on the sale of the security prior to maturity.

The above is assuming there is no default by the issuer, a circumstance under which the bondholder will typically suffer a loss.

The above is also assuming that the bond is bought at par, which is usually not the case.

- If a bond is bought at a discount (at a premium) it is because the coupon is lower (higher) than the prevailing required yield.
- As time goes by, and assuming the required yield does not change, the discount (premium) will amortize creating a capital gain (loss) that will bring the total return in line with the required yield (see next slide) as the carrying value evolves toward par.

Bond prices over time



For various reasons, a bondholder might sell a given bond prior to maturity. The holding period return will only equals the initial yield to maturity if the prevailing yield to maturity when the bond is sold matches the initial yield to maturity when the bond was bought. Since it is seldom the case, the real return earned over the holding period will diverge from the initial yield to maturity (i.e. better or worse).

- Higher interest rates lead to lower bond valuation, itself leading to a lower holding period return when the bond is sold, and vice versa;
- Since unanticipated changes in interest rates influence bond prices, holding period returns cannot be forecasted with any great degree of accuracy.

For performance evaluation, fixed income portfolios are periodically marked-to-market.

- The market price of each fixed income security is obtained (e.g. using Bloomberg);
- Using such prices and taking into consideration any intervening cash flows, the return of the portfolio is carefully calculated.

A fixed income security at its core is a contractual agreement between two parties under which one party (the investor or lender) agrees to provide funds against an undertaking by a second party (the issuer or borrower) to repay the funds plus interest.

- The issuer can only fulfill its undertaking if it has the required cash on hand to do so;
- With the exception of sovereigns having the ability to print their own currency, all issuers might find themselves unable to comply with their contractual agreements and default;
- Credit risk is the possibility that an issuer might default and that in turn such default results in investors earning lower returns than expected (and even to suffer losses);
- So, investors have an incentive to anticipate such consequences, and to require extra compensation from the issuer under the form of extra yield to compensate such credit risk.

$$\textit{Expected loss} = \textit{Default probability} \times \textit{Loss severity given default}$$

$$\textit{Required extra yield}\ \ (\textit{the default premium}) = \textit{Expected loss} = 0.02 \times 0.5 = 0.01$$

- Since a yield to maturity as calculated assumes no default, expected as well as realized returns are likely on average to be slightly smaller than the yield to maturity.

A credit rating, as issued by a credit rating agency upon an independent assessment of credit risk, is 'a symbol-based measure of the potential risk of default of a particular bond or issuer'.

		Moody's	S&P	Fitch
Investment Grade	High-Quality Grade	Aaa	AAA	AAA
		Aa1	AA+	AA+
		Aa2	AA	AA
		Aa3	AA–	AA–
	Upper-Medium Grade	A1	A+	A+
		A2	A	A
		A3	A–	A–
	Low-Medium Grade	Baa1	BBB+	BBB+
		Baa2	BBB	BBB
		Baa3	BBB–	BBB–

		Moody's	S&P	Fitch
Non-Investment Grade "Junk" or "High Yield"	Low Grade or Speculative Grade	Ba1	BB+	BB+
		Ba2	BB	BB
		Ba3	BB–	BB–
		B1	B+	B+
		B2	B	B
		B3	B–	B–
		Caa1	CCC+	CCC+
		Caa2	CCC	CCC
		Caa3	CCC–	CCC–
		Ca	CC	CC
		C	C	C
	Default	C	D	D

source: CFA Institute (2019)

Credit risk: default rates by credit rating category

Default rates increase as credit rating decreases (as intended for a ranking system), but are also influenced by the economic environment (e.g. 2001-2002 and 2008-2009 recessions).

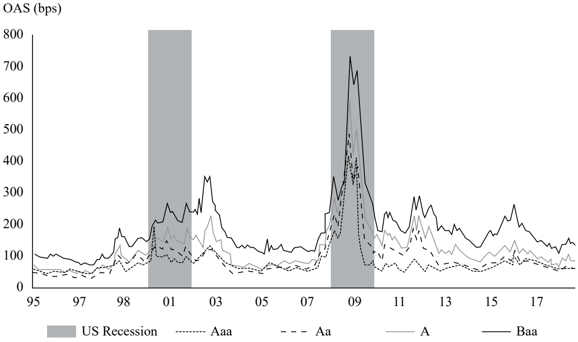
	AAA	AA	A	BBB	BB	B	CCC/C
1998	0.00	0.00	0.00	0.41	0.82	4.63	42.86
1999	0.00	0.17	0.18	0.20	0.95	7.29	33.33
2000	0.00	0.00	0.27	0.37	1.16	7.70	35.96
2001	0.00	0.00	0.27	0.34	2.96	11.53	45.45
2002	0.00	0.00	0.00	1.01	2.89	8.21	44.44
2003	0.00	0.00	0.00	0.23	0.58	4.07	32.73
2004	0.00	0.00	0.08	0.00	0.44	1.45	16.18
2005	0.00	0.00	0.00	0.07	0.31	1.74	9.09
2006	0.00	0.00	0.00	0.00	0.30	0.82	13.33
2007	0.00	0.00	0.00	0.00	0.20	0.25	15.24
2008	0.00	0.38	0.39	0.49	0.81	4.09	27.27
2009	0.00	0.00	0.22	0.55	0.75	10.94	49.46
2010	0.00	0.00	0.00	0.00	0.58	0.86	22.62
2011	0.00	0.00	0.00	0.07	0.00	1.67	16.30
2012	0.00	0.00	0.00	0.00	0.03	1.57	27.52
2013	0.00	0.00	0.00	0.00	0.10	1.64	24.50

source: CFA Institute (2019 using S&P data)

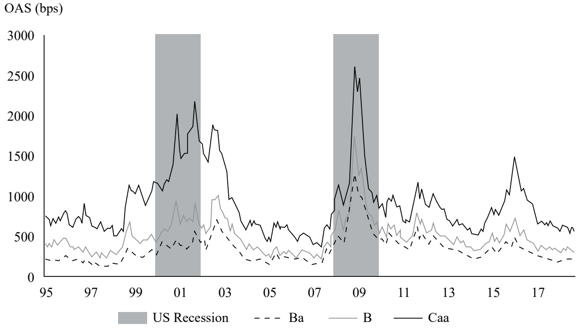
Credit risk: evidence of a time-varying default premium

The default premium (as proxied by the option-adjusted spread) increases as credit rating decreases (as expected), but is also influenced by the economic environment.

A. Investment-Grade Corporate Spreads



B. High-Yield Corporate Spreads



source: CFA Institute (2019 using Bloomberg data)

Yield determinants for fixed income securities

$$\begin{aligned} \text{Yield on sovereign bond} &= \text{Real risk free interest rate} \\ &+ \text{Expected inflation rate} \\ &+ \text{Maturity premium} \end{aligned}$$

$$\begin{aligned} \text{Yield on corporate bond} &= \text{Real risk free interest rate} \\ &+ \text{Expected inflation rate} \\ &+ \text{Maturity premium} \\ &+ \text{Liquidity premium} \\ &+ \text{Credit spread} \end{aligned}$$

$$\text{Yield spread} = \text{Liquidity premium} + \text{Credit spread}$$

The universe of fixed income securities

The universe of fixed income securities is significantly wider and more complex than the universe of equity securities.

- More issuers (in addition to publicly-listed firms: supranationals like the IMF or the EIB, sovereigns, private firms, public entities, structured finance entities, securitizations vehicles, etc.);
- Various maturities and even various currencies for each issuer;
- Various repayment and coupon structures as well as embedded options;
- Possibility of swapping fixed into floating, and vice versa;
- Multiple seniority levels and credit enhancements (like collateral, etc).

Given all of the above, there is a need to have the required analytics to adequately support the investment decision making process.

Learning Objectives covered

- L01 to L05

Concept checks

- Concept checks 1 to 7, and 9 (solutions provided at the end of the chapter).

Exercises

- The concepts and the example on next slide are enough assuming you have mastered doing such things in Excel.
- If you need to brush-up your Excel skills, I suggest looking carefully at the Excel slides following next slide.

Calculate the present value of a 4-year bond having an annual coupon of 8% when the required yield is 4%, 8% and 12%

$$B_{t=0@4\%} = \frac{8}{(1+.04)^1} + \frac{8}{(1+.04)^2} + \frac{8}{(1+.04)^3} + \frac{108}{(1+.04)^4} = 114.52$$

$$B_{t=0@8\%} = \frac{8}{(1+.08)^1} + \frac{8}{(1+.08)^2} + \frac{8}{(1+.08)^3} + \frac{108}{(1+.08)^4} = 100.00$$

$$B_{t=0@12\%} = \frac{8}{(1+.12)^1} + \frac{8}{(1+.12)^2} + \frac{8}{(1+.12)^3} + \frac{108}{(1+.12)^4} = 87.85$$

A single discount rate being applied to all cash flows when pricing a bond is called a Yield-To-Maturity (YTM).

Using Excel in finance

Excel is often used in finance (sometimes with catastrophic outcomes).

- To calculate a finance quantity (PV, FV, NPV, etc.), you can implement the close form solution in Excel (i.e. the formulas seen on previous slides) and/or use an Excel financial function and/or use a step-by-step tabular approach (i.e. three different ways).
- Using an Excel function is like using a black box, therefore risky.
- It is advisable to carefully implement two or even three different ways when calculating a finance quantity with Excel.

The Excel examples on the following slides are in the Excel file to be found on the course website. It is recommended to carefully review this Excel file alongside the slides.

The PV and FV functions in Excel can be used to calculate the PV and FV of various cash flows.

=PV(rate, nper, pmt, [fv], [type]) and

=FV(rate, nper, pmt, [pv], [type])

- rate: the interest rate per period
- nper: the number of periods
- pmt: the 'payment' (i.e. cash flows of same amount)
- fv (or pv): a cash flow which is occurring at the end of the series (or at the beginning)
- type: 0 (or omitted) if the cash flows are occurring at the end of the periods, or 1 if at the beginning of the periods like a rental or a lease

Calculate future values & present values with Excel

What is the future value of a \$10,000 investment held for 3 years at 5%?
=FV(rate, nper, pmt, [PV], [type])=FV(5%,3,0,-10000)=11,576

What is the present value of \$10,000 received 3 years from now at 5%?
=PV(rate, nper, pmt, [FV], [type])=PV(5%,3,0,10000)=-8,638

What is the present value of a \$10,000 perpetuity at 5%?
=PV(rate, nper, pmt, [FV], [type])=PV(5%,1000,10000,0)=-200,000

What is the present value of a 5-year \$10,000 annuity at 5%?
=PV(rate, nper, pmt, [FV], [type])=PV(5%,5,10000,0)=-43,295

What is the PV of a 5-year \$10,000 annuity at 5% growing at 2%?
=PV((1+5%)/(1+2%)-1,5,10000,0)/(1+2%)=-44,975

Other useful Excel financial functions

The RATE function in Excel can be used to calculate the required interest rate of various cash flows while the PMT function calculates the regular periodic cash flow given all other parameters.

=RATE(nper, pmt, pv, [fv], [type], [guess])

=PMT(rate, nper, pv, [fv], [type])

- rate: the interest rate per period
- nper: the number of periods
- pmt: the ‘payment’ (i.e. cash flows of same amount)
- fv (or pv): a cash flow occurring at the end of the series (or at the beginning)
- type: 0 (or omitted) if the cash flows are occurring at the end of the periods, or 1 if at the beginning of the periods like a rental or a lease
- guess: if the function does not converge use your own guess

What is the interest rate required for an initial investment of \$10,000 to grow to \$11,576 in 3 years?

=RATE(nper,pmt,pv,[fv],[type],[guess])=RATE(3,0,-10000,11576)=5%

The close form solution can be found with some algebra.

$$PV = \frac{C_T}{(1+r)^T} \rightarrow (1+r)^T = \frac{C_T}{PV} \rightarrow (1+r) = \left(\frac{C_T}{PV}\right)^{1/T} \rightarrow r = \left(\frac{C_T}{PV}\right)^{1/T} - 1$$

What is the payment required for a beginning of period annuity to grow to \$58,019 in 5 years at 5%?

=PMT(rate, nper, pv, [fv], [type])=PMT(5%,5,0,58019,1)=10,000

The close form solution can be found with some algebra, but corrected for beginning of period cash flow.

$$FV = PV(1+r)^T = C\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right](1+r)^T \rightarrow C = \frac{FV}{\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right](1+r)^{T+1}}$$

The NPER function in Excel can be used to calculate the number of periods given all other parameters.

=NPER(rate, pmt, pv, [fv], [type])

The close form solution for a single cash flow can be found with some algebra.

$$PV = \frac{C_T}{(1+r)^T} \rightarrow (1+r)^T = \frac{C_T}{PV} \rightarrow ln\left[(1+r)^T\right] = ln\left[\frac{C_T}{PV}\right] \rightarrow T = \frac{ln\left(\frac{C_T}{PV}\right)}{ln(1+r)}$$

What is the time required for an investment of \$10,000 to grow to \$11,576 at 5%?

=NPER(rate, pmt, pv, [fv], [type])=NPER(5%,0,-10000,11576)=3